

# 2D Trigonometry

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Name: \_\_\_\_\_

Date: \_\_\_\_\_

## At a glance

Pythagoras, SOH CAH TOA, elevation, depression and bearings, the sine and cosine rules, and  $\frac{1}{2}ab \sin C$ .

1. Pythagoras' theorem
2. Sin, cos and tan (SOH CAH TOA)
3. Elevation, depression and bearings
4. Sine rule and cosine rule
5. Area of any triangle

## 1. Pythagoras' theorem

☐ I revised this

- In a right-angled triangle:  $a^2 + b^2 = c^2$ , where  $c$  is the hypotenuse (opposite the right angle, the longest side).
- Finding the hypotenuse: square, **add**, square root. A shorter side: square, **subtract**, square root.
- Pythagorean triples to recognise: 3-4-5, 5-12-13, 8-15-17, 7-24-25 (and multiples like 6-8-10).

### Formulas

$$a^2 + b^2 = c^2$$

$c$  = hypotenuse

### Worked example

A ladder 6.5 m long leans against a wall with its foot 2.5 m from the wall. How high up the wall does it reach?

1. Hypotenuse 6.5:  $h^2 = 6.5^2 - 2.5^2$ .
2.  $= 42.25 - 6.25 = 36$ .

**6 m**

## 2. Sin, cos and tan (SOH CAH TOA)

☐ I revised this

- Label the sides from the angle: **Opposite**, **Adjacent** (next to the angle), **Hypotenuse**.
- Choose the ratio that uses the two sides involved (one known, one wanted).
- Finding an angle: use the inverse ( $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ ). Calculator in degrees.

|     | 0° | 30°                  | 45°                  | 60°                  | 90° |
|-----|----|----------------------|----------------------|----------------------|-----|
| sin | 0  | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1   |
| cos | 1  | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0   |
| tan | 0  | $\frac{\sqrt{3}}{3}$ | 1                    | $\sqrt{3}$           | —   |

### Formulas

$$\sin \theta = O/H$$

$$\cos \theta = A/H$$

$$\tan \theta = O/A$$

### Worked example

In a right-angled triangle the angle is  $52^\circ$  and the adjacent side is 8 cm. Find the opposite side.

1. O and A  $\rightarrow \tan$ .
2.  $O = 8 \times \tan 52^\circ = 10.239\dots$

**10.2 cm (3 s.f.)**

## 3. Elevation, depression and bearings

☐ I revised this

- **Angle of elevation:** measured up from the horizontal. **Angle of depression:** down from the horizontal.
- The angle of depression from A to B equals the angle of elevation from B to A (alternate angles).
- Bearings questions: draw north lines, mark the bearings, find a right-angled (or other) triangle.
- The shortest distance from a point to a line is the perpendicular distance.

### Worked example

From a point 20 m from the foot of a tower, the angle of elevation of the top is  $35^\circ$ . Find the height.

1.  $\tan 35^\circ = h \div 20$ .
2.  $h = 20 \times \tan 35^\circ = 14.0\dots$

**14.0 m (3 s.f.)**

## 4. Sine rule and cosine rule

☐ I revised this

- For triangles **without** a right angle. Label sides a, b, c opposite angles A, B, C.
- **Sine rule:** use when you know a side and its opposite angle (plus one more).
- **Cosine rule:** use with two sides and the angle between them (to find the third side) or all three sides (to find an angle).
- A negative cosine means an obtuse angle.  $\sin x = \sin(180^\circ - x)$ : the sine rule may give an obtuse answer (the ambiguous case).

### Formulas

|                                    |                     |
|------------------------------------|---------------------|
| $a/\sin A = b/\sin B = c/\sin C$   | sine rule           |
| $a^2 = b^2 + c^2 - 2bc \cos A$     | cosine rule (side)  |
| $\cos A = (b^2 + c^2 - a^2) / 2bc$ | cosine rule (angle) |

### Worked example

Triangle PQR: PQ = 6 cm, PR = 9 cm, angle P =  $50^\circ$ . Find QR.

1.  $QR^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 50^\circ$ .
2.  $= 117 - 69.42 = 47.58$ .

**QR = 6.90 cm (3 s.f.)**

## 5. Area of any triangle

☐ I revised this

- Area =  $\frac{1}{2}ab \sin C$ , where C is the angle **between** sides a and b.
- Use it backwards to find an angle or a side when the area is given.

### Formulas

$$\text{Area} = \frac{1}{2}ab \sin C$$

### Worked example

A triangle has sides 8 cm and 10 cm with  $30^\circ$  between them. Find its area.

1.  $\frac{1}{2} \times 8 \times 10 \times \sin 30^\circ$ .

2.  $= 40 \times 0.5$ .

**20 cm<sup>2</sup>**

### Common mistakes

| Don't write                                                             | Write instead                                    |
|-------------------------------------------------------------------------|--------------------------------------------------|
| ✗ Using SOH CAH TOA without a right angle                               | ✓ Use the sine or cosine rule                    |
| ✗ Forgetting to square root at the end of Pythagoras or the cosine rule | ✓ $c^2 = 49 \rightarrow c = 7$                   |
| ✗ Adding in Pythagoras when finding a shorter side                      | ✓ Shorter side: subtract                         |
| ✗ Calculator in radians                                                 | ✓ Set it to degrees                              |
| ✗ Giving only the acute angle when the triangle is obtuse               | ✓ The obtuse angle is $180^\circ - \text{acute}$ |

### In the exam

- Draw and label the triangle, marking O, A, H or a, b, c, A, B, C.
- Keep full calculator values between steps; round only at the end (angles to 1 d.p.).
- Non-calculator paper: use the exact-value table.

### My notes

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