

3D Trigonometry

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Name: _____

Date: _____

At a glance

Pythagoras and trigonometry in cuboids, pyramids and prisms, and the angle between a line and a plane.

1. Lengths in 3D
2. The angle between a line and a plane
3. Pyramids
4. Prisms and wedges

1. Lengths in 3D

☐ I revised this

- Find a right-angled triangle that contains the length you want, then use Pythagoras.
- Usually two steps: the diagonal of the base first, then the vertical triangle.
- Space diagonal of a cuboid $a \times b \times c$: $\sqrt{a^2 + b^2 + c^2}$.

Formulas

$$d = \sqrt{a^2 + b^2 + c^2}$$

space diagonal of a cuboid

Worked example

A box is 12 cm long, 4 cm wide and 3 cm high. Find the length of the longest rod that fits inside.

1. Base diagonal: $\sqrt{144 + 16} = \sqrt{160}$.
2. Space diagonal: $\sqrt{160 + 9} = \sqrt{169}$.

13 cm

2. The angle between a line and a plane

☐ I revised this

- Drop a perpendicular from the top of the line to the plane; the foot gives the line's **projection**.
- The angle is between the line and its projection — not the vertical.
- The right-angled triangle is: line (hypotenuse), projection (adjacent), vertical height (opposite).

How to do it

1. Identify the line and the plane.
2. Find the projection of the line on the plane.
3. Draw the right-angled triangle flat on its own.
4. Use SOH CAH TOA.

Worked example

A cuboid is 6 cm by 8 cm by 5 cm high. Find the angle between the space diagonal and the base.

1. Base diagonal (projection) = 10 cm.
2. $\tan \theta = 5 \div 10$.

26.6°

3. Pyramids

☐ I revised this

- The vertical height goes from the apex to the centre of the base.
- For a square or rectangular base, the centre is the midpoint of the base diagonal.
- The angle between a sloping edge and the base uses half the base diagonal; the angle between a sloping face and the base uses half the base width.

Worked example

A square-based pyramid has base $8\text{ cm} \times 8\text{ cm}$ and vertical height 6 cm . Find the angle between a sloping edge and the base.

1. Half the base diagonal $= 4\sqrt{2} = 5.657\text{ cm}$.
2. $\tan \theta = 6 \div 5.657$.

46.7°

4. Prisms and wedges

☐ I revised this

- Look for the right angles between the faces.
- Redraw each triangle you need as a flat 2D triangle with its lengths.
- You may need the sine or cosine rule in a non-right-angled face.

Worked example

A wedge has a horizontal base 12 cm by 5 cm and a vertical back edge 9 cm high above one 5 cm side. Find the length from a front bottom corner to the opposite top corner.

1. Base diagonal: $\sqrt{(144 + 25)} = 13$.
2. Then with the 9 cm height: $\sqrt{(169 + 81)} = \sqrt{250}$.

15.8 cm (3 s.f.)

Common mistakes

Don't write	Write instead
 Using the base side instead of the base diagonal	 The projection of a space diagonal is the base diagonal
 Measuring the angle from the vertical	 The angle with a plane is measured from the plane
 Rounding $\sqrt{160}$ to 12.6 early	 Keep it exact or keep all digits

In the exam

- Draw every triangle you use separately and label it.
- Name the angle with letters (e.g. angle AGE) so the examiner can follow.
- Give angles to 1 decimal place.

My notes