

Algebra

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Expressions, expanding, linear equations, forming equations, changing the subject, algebraic fractions and quadratics.

1. Expressions and substitution
2. Expanding brackets
3. Solving linear equations
4. Forming expressions and equations
5. Changing the subject
6. Algebraic fractions
7. Quadratic equations

1. Expressions and substitution

☐ I revised this

- Collect **like terms** only: $3x + 2y - x = 2x + 2y$; x^2 and x are not like terms.
- Substitute with brackets round negative numbers: if $a = -2$, then $a^2 = (-2)^2 = 4$ and $3a = -6$.
- Write terms in a standard way: $3 \times a \times a \times b = 3a^2 b$.

Worked example

Find the value of $2p^2 - 5q$ when $p = 4$ and $q = -3$.

1. $2 \times 4^2 = 2 \times 16 = 32$.
2. $5 \times (-3) = -15$.
3. $32 - (-15) = 32 + 15$.

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2. Expanding brackets

☐ I revised this

- Single bracket: multiply every term inside. $-3(2x - 5) = -6x + 15$ (watch the signs).
- Two brackets: every term in the first $\times$ every term in the second (FOIL or a grid), then collect like terms.
- Three brackets: expand two of them first, then multiply the result by the third.
- Perfect squares: $(a + b)^2 = a^2 + 2ab + b^2$ — never just $a^2 + b^2$.

Formulas

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Worked example

Expand and simplify $(x + 2)(x - 3)(2x + 1)$.

1. $(x + 2)(x - 3) = x^2 - x - 6$.
2. $(x^2 - x - 6)(2x + 1) = 2x^3 + x^2 - 2x^2 - x - 12x - 6$.
3. Collect like terms.

$2x^3 - x^2 - 13x - 6$

3. Solving linear equations

☐ I revised this

- Do the same to both sides until the letter is on its own.
- Brackets: expand first. Letters on both sides: collect them on the side with more.
- Fractions: multiply every term by the lowest common denominator first.
- Algebraic denominators: $3/(x - 1) = 2/(x + 2) \rightarrow$ cross-multiply: $3(x + 2) = 2(x - 1) \rightarrow x = -8$.

How to do it

1. Clear fractions ($\times$ LCD).
2. Expand brackets.
3. Collect x terms on one side, numbers on the other.
4. Divide by the number in front of x .
5. Check by substituting.

Worked example

Solve $(x + 1)/3 - (x - 2)/4 = 2$.

1. Multiply every term by 12: $4(x + 1) - 3(x - 2) = 24$.
2. $4x + 4 - 3x + 6 = 24$.
3. $x + 10 = 24$.

$x = 14$

Exam tip: Put the second numerator in brackets: $-3(x - 2) = -3x + 6$, not $-3x - 6$.

4. Forming expressions and equations

☐ I revised this

- Turn words into algebra: '3 more than x ' = $x + 3$, 'twice x ' = $2x$, ' x squared' = x^2 .
- Perimeter adds all the sides; area multiplies; angles in a triangle add to 180° .
- Define your letter clearly, form the equation, solve it, then answer the question asked.

Worked example

A rectangle's length is 3 cm more than its width. Its perimeter is 34 cm. Find its area.

1. Width w , length $w + 3$.
2. $2(w + w + 3) = 34 \rightarrow 4w + 6 = 34$.
3. $w = 7$, length 10.

70 cm^2

5. Changing the subject

☐ I revised this

- Undo the operations on the new subject in reverse order, doing the same to both sides.
- Square $\leftrightarrow$ square root; $\times \leftrightarrow \div$; $+$ $\leftrightarrow$ $-$.
- If the subject appears twice: collect those terms on one side, **factorise** it out, then divide.

When the subject appears twice

1. Clear fractions and roots.
2. Expand any brackets.
3. Get every term with the subject on one side.
4. Factorise the subject out.
5. Divide.

Worked example

Make t the subject of $3t + a = bt - 5$.

1. Collect t terms: $3t - bt = -5 - a$.
2. Factorise: $t(3 - b) = -5 - a$.
3. Divide by $(3 - b)$.

$$t = (a + 5)/(b - 3)$$

Exam tip: Example with a root: $V = \frac{1}{3}\pi r^2 h$ gives $r = \sqrt{(3V/(\pi h))}$.

6. Algebraic fractions

☐ I revised this

- **Simplify:** factorise top and bottom completely, then cancel whole brackets (never single terms).
- **Add/subtract:** common denominator (often the product of the bottoms), then simplify the top.
- **Multiply/divide:** like number fractions — flip the second when dividing; factorise and cancel first.

Worked example

Write $2/(x + 1) + 3/(x - 2)$ as a single fraction.

1. Common denominator $(x + 1)(x - 2)$.
2. Top: $2(x - 2) + 3(x + 1) = 2x - 4 + 3x + 3$.
3. $= 5x - 1$.

$$(5x - 1)/((x + 1)(x - 2))$$

Exam tip: You can cancel $(x + 3)$ from top and bottom, but not the x in $(x + 3)/x$.

7. Quadratic equations

☐ I revised this

- Rearrange to $ax^2 + bx + c = 0$ first.
- **Factorise** when it works; each bracket = 0 gives a solution.
- Use the **formula** when the question asks for decimals ('2 d.p.', '3 s.f.').
- **Completing the square:** $x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$. It gives the turning point and exact solutions.
- If $b^2 - 4ac$ is negative there are no real solutions.

Formulas

$$x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$$

the quadratic formula

$$x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$$

completing the square

Worked example

Solve $2x^2 - 5x - 1 = 0$, giving your answers to 2 decimal places.

1. $a = 2$, $b = -5$, $c = -1$.
2. $x = (5 \pm \sqrt{(25 + 8)}) \div 4 = (5 \pm \sqrt{33}) \div 4$.
3. $\sqrt{33} = 5.7446\dots$

$$x = 2.69 \text{ or } x = -0.19$$

Exam tip: Completing the square on $x^2 - 8x + 11$ gives $(x - 4)^2 - 5$: minimum point $(4, -5)$, and solutions $x = 4 \pm \sqrt{5}$.

Common mistakes

Don't write	Write instead
$\times (x + 3)^2 = x^2 + 9$	$\checkmark$ There is a middle term: $x^2 + 6x + 9$
$\times -3(x - 2) = -3x - 6$	$\checkmark$ Negative times negative: $-3x + 6$
$\times x(x + 3) = 40$, so $x = 40$ or $x + 3 = 40$	$\checkmark$ Only '= 0' lets you split: $x^2 + 3x - 40 = 0$
$\times$ Cancelling x in $(x + 3)/x$ to get 3	$\checkmark$ Only whole factors cancel
$\times$ Dividing only part of the top by $2a$ in the formula	$\checkmark$ Divide the whole of $-b \pm \sqrt{b^2 - 4ac}$ by $2a$

In the exam

- Show the substitution into the quadratic formula — it is worth a mark on its own.
- Give both solutions of a quadratic unless the context rules one out (a negative length).
- Check a linear equation's answer by substituting it back in.

My notes

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