

# Arithmetics

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Name: \_\_\_\_\_

Date: \_\_\_\_\_

## At a glance

The number toolkit used in every other chapter: kinds of number, BIDMAS, fractions, decimals and recurring decimals.

1. Types of number
2. Order of operations and negative numbers
3. Fractions
4. Decimals and ordering
5. Halfway values and differences
6. Recurring decimals to fractions

## 1. Types of number

☐ I revised this

- **Natural numbers:** 1, 2, 3, ... **Integers:** whole numbers including 0 and negatives (... , -2, -1, 0, 1, 2, ...).
- **Prime:** exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... 1 is **not** prime; 2 is the only even prime.
- **Factor** of 12: divides into it (1, 2, 3, 4, 6, 12). **Multiple** of 12: in its times table (12, 24, 36, ...).
- **Rational:** can be written as a fraction  $a/b$  (all integers, terminating and recurring decimals). **Irrational:** cannot —  $\pi$ ,  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$  and the square root of any non-square number.
- **Reciprocal** of  $x$  is  $1/x$ : reciprocal of 4 is  $1/4$ , of  $2/5$  is  $5/2$ . A number times its reciprocal is 1.

| n     | 1 | 2 | 3  | 4  | 5   | 6   | 7   | 8   | 9   | 10   |
|-------|---|---|----|----|-----|-----|-----|-----|-----|------|
| $n^2$ | 1 | 4 | 9  | 16 | 25  | 36  | 49  | 64  | 81  | 100  |
| $n^3$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 |

## Worked example

From -3, 0,  $1/2$ ,  $\sqrt{9}$ ,  $\sqrt{5}$ , 7 and 0.333..., list (a) the irrational numbers, (b) the prime numbers.

1.  $\sqrt{9} = 3$  and  $0.333... = 1/3$ , so both are rational.
2.  $\sqrt{5}$  is not a whole number and does not recur: irrational.
3. Primes are positive whole numbers with exactly two factors:  $\sqrt{9} = 3$  and 7.

(a)  $\sqrt{5}$  (b)  $\sqrt{9}$  and 7

**Exam tip:** Also know  $11^2 = 121$ ,  $12^2 = 144$ ,  $13^2 = 169$ ,  $14^2 = 196$ ,  $15^2 = 225$  for the non-calculator paper.

## 2. Order of operations and negative numbers

☐ I revised this

- **BIDMAS:** Brackets → Indices → Division and Multiplication (left to right) → Addition and Subtraction (left to right).
- Two signs next to each other: same signs give +, different signs give -. So  $5 - (-3) = 8$  and  $4 \times (-2) = -8$ .
- $(-3)^2 = 9$  but  $-3^2 = -9$ : without brackets only the 3 is squared.
- A fraction line acts like brackets:  $(8 + 4) \div (5 - 2) = 4$ . On a calculator, put brackets round the top and the bottom.

## Order to work in

1. Work out anything in brackets.
2. Then powers and roots.
3. Then  $\times$  and  $\div$  from left to right.
4. Finally + and - from left to right.

### Worked example

Work out  $(-3)^2 - 4 \times (5 - 7)$ .

1. Brackets:  $5 - 7 = -2$ .
2. Power:  $(-3)^2 = 9$ .
3. Multiply:  $4 \times (-2) = -8$ .
4.  $9 - (-8) = 9 + 8$ .

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## 3. Fractions

☐ I revised this

- To add or subtract, use a **common denominator** (the LCM of the bottoms), then add the tops.
- To multiply, multiply tops and bottoms (cancel first to keep numbers small).
- To divide, **flip the second fraction and multiply**:  $a/b \div c/d = a/b \times d/c$ .
- Change mixed numbers to improper fractions before multiplying or dividing:  $2\frac{3}{4} = 11/4$ .
- Fraction of an amount: divide by the bottom, multiply by the top.  $3/8$  of 240 =  $240 \div 8 \times 3 = 90$ .
- Always give the answer in its simplest form.

### Formulas

|                                 |                                |
|---------------------------------|--------------------------------|
| $a/b + c/d = (ad + bc)/bd$      | then simplify                  |
| $a/b \times c/d = ac/bd$        | multiply across                |
| $a/b \div c/d = a/b \times d/c$ | flip the second, then multiply |

### Worked example

Work out  $2\frac{3}{4} - 1\frac{5}{6}$ .

1. Improper fractions:  $11/4 - 11/6$ .
2. Common denominator 12:  $33/12 - 22/12$ .
3.  $= 11/12$ .

11/12

**Exam tip:** On the non-calculator paper write every step: the method marks need the common denominator to be seen.

## 4. Decimals and ordering

☐ I revised this

- Multiplying decimals: ignore the points, multiply, then give the answer as many decimal places as the question had in total.  $0.3 \times 0.04 = 0.012$ .
- Dividing by a decimal: multiply both numbers by 10, 100, ... until you divide by a whole number.  $4.8 \div 0.06 = 480 \div 6 = 80$ .
- Fraction  $\rightarrow$  decimal: divide the top by the bottom.  $3/8 = 0.375$ .
- To order a mixture of fractions, decimals and percentages, change them all to decimals (to 3 or 4 places).
- Symbols: = equal,  $\neq$  not equal, > greater than, < less than,  $\geq$  greater than or equal to,  $\leq$  less than or equal to.

### Worked example

Write in order, smallest first: 0.7,  $2/3$ , 68%, 0.67.

1.  $2/3 = 0.6667$ ,  $68\% = 0.68$ .
2. Compare: 0.6667, 0.67, 0.68, 0.7.

2/3, 0.67, 68%, 0.7

## 5. Halfway values and differences

☐ I revised this

- The number **halfway** between a and b is  $(a + b) \div 2$ .
- The **difference** between two numbers is the larger minus the smaller; across zero, add the distances: from  $-12$  to  $9$  is  $12 + 9 = 21$ .
- Temperature questions: a rise from  $-7^\circ\text{C}$  to  $5^\circ\text{C}$  is 12 degrees; a fall of 9 degrees from  $4^\circ\text{C}$  gives  $-5^\circ\text{C}$ .

### Formulas

$$\text{halfway} = (a + b) \div 2$$

works with negatives and decimals

### Worked example

Find the number halfway between  $-2.6$  and  $4.2$ .

1. Add:  $-2.6 + 4.2 = 1.6$ .

2. Halve:  $1.6 \div 2$ .

**0.8**

## 6. Recurring decimals to fractions

☐ I revised this

- A dot over a digit means it repeats:  $0.4' = 0.444\dots$ ,  $0.27' = 0.272727\dots$
- Every recurring decimal is a fraction. One repeating digit  $\rightarrow$  over 9; two repeating digits  $\rightarrow$  over 99.
- If some digits do not repeat ( $0.3666\dots$ ), multiply twice so the repeating parts line up, then subtract.

### How to do it

1. Let  $x$  = the decimal.
2. Multiply by 10 (one repeating digit) or 100 (two) so the repeating part lines up.
3. Subtract to remove the repeating part.
4. Divide and simplify.

| Fraction | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{9}$ | $\frac{1}{11}$ | $\frac{1}{6}$ |
|----------|---------------|---------------|---------------|----------------|---------------|
| Decimal  | $0.3'$        | $0.6'$        | $0.1'$        | $0.09'$        | $0.16'$       |

### Worked example

Write  $0.3666\dots$  as a fraction in its simplest form.

1.  $x = 0.3666\dots$ ,  $10x = 3.666\dots$ ,  $100x = 36.666\dots$

2.  $100x - 10x: 90x = 33$ .

3.  $x = 33/90$ .

**$\frac{11}{30}$**

**Exam tip:** 'Show that  $0.27 = 3/11$ ' needs the algebra ( $x = \dots$ ,  $100x = \dots$ , subtract). Just dividing 3 by 11 gets no marks.

## Common mistakes

| Don't write                                                                                                                 | Write instead                                                                                                                                    |
|-----------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
|  Calling 1 a prime number                  |  1 has only one factor, so it is not prime                      |
|  $3 + 4 \times 2 = 14$                     |  Multiply first: $3 + 8 = 11$                                   |
|  $-3^2 = 9$                                |  Only the 3 is squared: $-9$ . $(-3)^2 = 9$                     |
|  $1/2 + 1/3 = 2/5$                         |  Common denominator: $3/6 + 2/6 = 5/6$                          |
|  $2^{3/4} \times 2 = 4^{3/4}$              |  Improper fraction first: $11/4 \times 2 = 11/2 = 5\frac{1}{2}$ |
|  Ordering 0.7 before 0.67 because $7 < 67$ |  Compare place by place: $0.70 > 0.67$                          |

## In the exam

- Non-calculator questions want every step shown — a correct answer with no working may lose marks.
- Give fractions in their simplest form unless the question asks otherwise.
- Check an answer is sensible: dividing by a number less than 1 makes it bigger.

## My notes

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