

Differentiation

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Differentiating ax^n , gradients and tangents, and turning points with maximum or minimum.

1. Differentiating
2. Gradients and tangents
3. Turning points

1. Differentiating

☐ I revised this

- The derivative dy/dx gives the **gradient** of the curve at any point.
- For ax^n : multiply by the power, then take one off the power.
- A term in x differentiates to a number ($7x \rightarrow 7$); a constant differentiates to 0.
- Differentiate a sum term by term.

Formulas

$$y = ax^n \rightarrow dy/dx = nax^{n-1}$$

$$y = kx \rightarrow dy/dx = k$$

$$y = c \rightarrow dy/dx = 0$$

Worked example

Differentiate $y = x^4 - 3x^2 + 7x - 2$.

1. $x^4 \rightarrow 4x^3$.

2. $-3x^2 \rightarrow -6x$.

3. $7x \rightarrow 7$; $-2 \rightarrow 0$.

$$dy/dx = 4x^3 - 6x + 7$$

2. Gradients and tangents

☐ I revised this

- Gradient at a point: differentiate, then substitute the x value.
- Where is the gradient k ? Set $dy/dx = k$ and solve.
- Equation of the tangent at (x_1, y_1) : $m = dy/dx$ there, then $y - y_1 = m(x - x_1)$.

Formulas

$$y - y_1 = m(x - x_1)$$

tangent through (x_1, y_1)

Worked example

Find the equation of the tangent to $y = x^2 + 3x$ at $x = 1$.

1. $y = 1 + 3 = 4$ at $x = 1$.

2. $dy/dx = 2x + 3 = 5$ at $x = 1$.

3. $y - 4 = 5(x - 1)$.

$$y = 5x - 1$$

3. Turning points

☐ I revised this

- At a turning (stationary) point the gradient is 0: solve $dy/dx = 0$.
- Find y by substituting x into the **original** equation.
- Maximum or minimum? Second derivative d^2y/dx^2 : negative $\rightarrow$ maximum, positive $\rightarrow$ minimum. Or check the gradient just either side.
- Real-life problems: maximise area, profit, volume by finding where the derivative is 0.

How to do it

1. Differentiate.
2. Set $dy/dx = 0$ and solve for x .
3. Substitute into y for the coordinates.
4. Use d^2y/dx^2 (or the gradient either side) to decide max or min.

Worked example

Find the turning points of $y = x^3 - 12x$ and say which is a maximum.

1. $dy/dx = 3x^2 - 12 = 0 \rightarrow x = \pm 2$.
2. $x = 2$: $y = 8 - 24 = -16$. $x = -2$: $y = -8 + 24 = 16$.
3. $d^2y/dx^2 = 6x$: at $x = 2$ it is 12 (minimum); at $x = -2$ it is -12 (maximum).

Maximum $(-2, 16)$; minimum $(2, -16)$

Common mistakes

Don't write	Write instead
 Differentiating $5x^3$ to $5x^2$	 Multiply by the power too: $15x^2$
 Leaving the constant in the derivative	 Constants differentiate to 0
 Finding y at a turning point from dy/dx	 Substitute into the original equation
 $7x$ differentiates to $7x^0$ left as $7x$	 $7x \rightarrow 7$

In the exam

- Write $dy/dx = \dots$ on its own line before substituting.
- State the coordinates of turning points, not just x .
- Say clearly 'maximum' or 'minimum' with your reason.

My notes