Functions

Quick revision notes

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At a glance

Function notation, inverses, composite functions, domain, range and asymptotes.

1. Function notation
2. Inverse functions
3. Composite functions
4. Domain, range and asymptotes

1. Function notation

I revised this

- f(x) = 3x – 4 is a rule: f(2) means put 2 in place of x: f(2) = 2.
- Solve f(x) = 11 by writing 3x – 4 = 11.
- f(x + 1) means replace x by (x + 1): 3(x + 1) – 4 = 3x – 1.
- 2f(x) = 2(3x – 4) = 6x – 8 but f(2x) = 6x – 4.

Worked example

g(x) = x^2 – 2x. Find g(–3) and solve g(x) = 8.

1. g(–3) = 9 + 6 = 15.
2. x^2 – 2x – 8 = 0 → (x – 4)(x + 2) = 0.

g(–3) = 15; x = 4 or x = –2

2. Inverse functions

I revised this

- f^{-1} undoes f: if f(3) = 11 then f^{-1}(11) = 3.
- f^{-1}(x) is not 1/f(x).

How to do it

1. Write y = f(x).
2. Swap x and y.
3. Make y the subject.
4. Write f^{-1}(x) = …

Worked example

f(x) = (2x – 5)/3. Find f^{-1}(x).

1. y = (2x – 5)/3; swap: x = (2y – 5)/3.
2. 3x = 2y – 5.
3. 2y = 3x + 5.

f^{-1}(x) = (3x + 5)/2

Exam tip: Check: f^{-1}(f(x)) must give x back. Try a number.

3. Composite functions

I revised this

- fg(x) means f(g(x)): **do g first**, then f.
- Replace the x in f with the whole expression g(x), in brackets.
- ff(x) means apply f twice. fg and gf are usually different.

Worked example

$f(x) = 2x + 1$ and $g(x) = x^2$. Find $fg(x)$, $gf(x)$ and $fg(3)$.

1. $fg(x) = f(x^2) = 2x^2 + 1$.
2. $gf(x) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$.
3. $fg(3) = 2 \times 9 + 1 = 19$.

$fg(x) = 2x^2 + 1$, $gf(x) = 4x^2 + 4x + 1$, $fg(3) = 19$

4. Domain, range and asymptotes

☐ I revised this

- **Domain**: the x values allowed. Leave out any value that makes a denominator zero.
- **Range**: the y values you get out. x^2 is never negative, so the range of $f(x) = x^2$ is $f(x) \geqslant 0$.
- $y = k/(x – a) + b$ has asymptotes $x = a$ (vertical) and $y = b$ (horizontal).
- $y = a^x + b$ has the horizontal asymptote $y = b$.

Worked example

$f(x) = 3/(x – 2) + 1$. State the value excluded from the domain and the asymptotes.

1. $x – 2 = 0$ when $x = 2$.
2. As x gets large, $3/(x – 2) \to 0$, so $y \to 1$.

$x = 2$ excluded; asymptotes $x = 2$ and $y = 1$

Common mistakes

Don't write	Write instead
✗ $fg(2)$: working out f first	✓ fg means $f(g(2))$: do g first
✗ $f^{-1}(x) = 1/f(x)$	✓ The inverse undoes f; it is not the reciprocal
✗ $gf(x) = 2x + 1^2$	✓ Use brackets: $(2x + 1)^2$
✗ $2f(x)$ and $f(2x)$ treated as the same	✓ $2f(x)$ doubles the output; $f(2x)$ doubles the input

In the exam

- Use brackets every time you substitute an expression.
- Check an inverse with a number: $f(2)$ then f^{-1} of the answer should give 2.
- For 'solve $ff(x) = \ldots$' find $ff(x)$ as an expression first.

My notes