Hcf Lcm

Quick revision notes

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At a glance

Prime factors, HCF and LCM, the problems they solve, and using primes for roots.

1. Prime factorisation
2. HCF and LCM
3. Word problems
4. Square roots, cube roots and the smallest n

1. Prime factorisation

I revised this

- Every whole number above 1 can be written as a product of primes in exactly one way.
- Use a factor tree or keep dividing by the smallest prime (2, 3, 5, 7, 11, …).
- Write the answer in index form: $360 = 2^3 \times 3^2 \times 5$.

Repeated division

1. Divide by 2 until you can't.
2. Then by 3, then 5, then 7…
3. Stop at 1; collect equal primes as powers.

Worked example

Write 360 as a product of its prime factors.

1. $360 \div 2 = 180$, $\div 2 = 90$, $\div 2 = 45$.
2. $45 \div 3 = 15$, $\div 3 = 5$, $\div 5 = 1$.
3. Primes used: 2, 2, 2, 3, 3, 5.

$2^3 \times 3^2 \times 5$

2. HCF and LCM

I revised this

- **HCF** (highest common factor): the biggest number that divides into both.
- **LCM** (lowest common multiple): the smallest number both divide into.
- From prime factors: HCF = primes they **share**, lowest power of each. LCM = **every** prime, highest power of each.
- A Venn diagram of the prime factors works too: HCF = the overlap; LCM = everything in the diagram.

Formulas

HCF	shared primes, lowest powers
LCM	all primes, highest powers
HCF × LCM = a × b	for two numbers a and b

Worked example

Find the HCF and LCM of 84 and 120.

1. $84 = 2^2 \times 3 \times 7$; $120 = 2^3 \times 3 \times 5$.
2. HCF: $2^2 \times 3 = 12$.
3. LCM: $2^3 \times 3 \times 5 \times 7 = 840$.

HCF 12, LCM 840

3. Word problems

I revised this

- 'Together again', 'at the same time', 'smallest number of each' → **LCM**.
- 'Largest possible', 'cut into equal pieces', 'share into identical groups' → **HCF**.
- Algebraic HCF: HCF of the numbers × the lowest power of each shared letter. HCF of $12x^{2y}$ and $18xy^3$ is 6xy.

Worked example

Two lighthouses flash every 12 s and every 20 s. They flash together at 9:00. When do they next flash together?

1. Together again → LCM.
2. $12 = 2^2 \times 3$, $20 = 2^2 \times 5$, LCM $= 2^2 \times 3 \times 5 = 60$.

60 s later, at 9:01

4. Square roots, cube roots and the smallest n

I revised this

- A **square** number has every prime to an even power; a **cube** has every power a multiple of 3.
- $\sqrt{}$: halve every power. $\sqrt[3]{}$: divide every power by 3. $\sqrt{(2^2 \times 3^2 \times 7^2)} = 2 \times 3 \times 7 = 42$.
- Smallest number to multiply by to make a square: supply whatever makes every power even.

Worked example

Find the smallest whole number n such that 120n is a cube number.

1. $120 = 2^3 \times 3 \times 5$.
2. For a cube every power must be a multiple of 3.
3. Need $3^2 \times 5^2$ more.

n = 225

Common mistakes

Don't write	Write instead
✗ LCM of 18 and 24 = 18 × 24 = 432	✓ That is a common multiple, but the lowest is 72
✗ Writing 2 × 2 × 2 × 3 × 3 × 5 when 'index form' is asked	✓ Write $2^3 \times 3^2 \times 5$
✗ Including 1 in a prime factorisation	✓ 1 is not prime — leave it out
✗ Taking the highest powers for the HCF	✓ HCF uses the LOWEST powers of shared primes

In the exam

- Always check the HCF divides both numbers and both numbers divide the LCM.
- In word problems, say which one you are finding (HCF or LCM) before calculating.
- Show the prime factorisations — they earn method marks even if a multiplication slips.

My notes