

# Indices

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Name: \_\_\_\_\_

Date: \_\_\_\_\_

## At a glance

Powers and roots: the four laws, zero/negative/fractional powers, algebra with powers, and index equations.

1. The laws of indices
2. Zero, negative and fractional powers
3. Indices in algebra
4. Equations with the unknown in the power

## 1. The laws of indices

☐ I revised this

- The **index** (power) says how many times a number is multiplied by itself:  $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ .
- The laws only work when the **base is the same**:  $2^3 \times 2^4 = 2^7$ , but  $2^3 \times 3^4$  cannot be combined.
- Numbers in front are dealt with normally:  $3x^2 \times 4x^5 = 12x^7$ .

### Formulas

|                            |                                          |
|----------------------------|------------------------------------------|
| $a^m \times a^n = a^{m+n}$ | multiply $\rightarrow$ add the powers    |
| $a^m \div a^n = a^{m-n}$   | divide $\rightarrow$ subtract the powers |
| $(a^m)^n = a^{mn}$         | power of a power $\rightarrow$ multiply  |
| $(ab)^n = a^n b^n$         | everything inside is raised              |

### Worked example

Simplify  $2^5 \times 2^3 \div 2^4$  and find its value.

1. Add:  $5 + 3 = 8$ .
2. Subtract:  $8 - 4 = 4$ .
3.  $2^4 = 16$ .

**$2^4 = 16$**

## 2. Zero, negative and fractional powers

☐ I revised this

- $a^0 = 1$  for any  $a$  except 0:  $7^0 = 1$ ,  $(-5)^0 = 1$ .
- A **negative** power means 'one over':  $a^{-n} = 1/a^n$ . It never makes the answer negative:  $2^{-3} = 1/8$ .
- A fraction raised to a negative power flips:  $(2/3)^{-2} = (3/2)^2 = 9/4$ .
- A **fractional** power is a root:  $a^{1/2} = \sqrt{a}$ ,  $a^{1/3} = \sqrt[3]{a}$ , and  $a^{m/n} = (\sqrt[n]{a})^m$  — do the root first.
- Negative fractional: do all three — flip, root, power.  $8^{-2/3} = 1/(\sqrt[3]{8})^2 = 1/4$ .

### For $a^{-m/n}$

1. Negative sign? Flip the number (write 1 over it).
2. Bottom of the fraction? Take that root.
3. Top of the fraction? Raise to that power.

## Formulas

|                             |                        |
|-----------------------------|------------------------|
| $a^0 = 1$                   |                        |
| $a^{-n} = 1/a^n$            | one over               |
| $a^{1/n} = \sqrt[n]{a}$     | root                   |
| $a^{m/n} = (\sqrt[n]{a})^m$ | root first, then power |

### Worked example

Find the value of  $(27/8)^{-2/3}$ .

1. Flip:  $(8/27)^{2/3}$ .
2. Cube root:  $\sqrt[3]{8} = 2$  and  $\sqrt[3]{27} = 3$ , giving  $2/3$ .
3. Square:  $(2/3)^2$ .

**$4/9$**

**Exam tip:** Know the powers of 2 (2, 4, 8, 16, 32, 64, 128), 3 (3, 9, 27, 81, 243) and 5 (5, 25, 125, 625) by heart.

## 3. Indices in algebra

☐ I revised this

- Deal with the numbers and each letter separately.
- $(3x^2)^3 = 3^3 \times x^6 = 27x^6$  — the number in front is cubed too.
- $12a^5 \div 3a^{-2} = 4a^{5-(-2)} = 4a^7$ .
- $(2x^{3/5})^2 = 4x^{6/25}$ : square the top and the bottom.

### Worked example

Simplify  $(2x^3 y)^4 \div 8x^5 y^2$ .

1. Expand the bracket:  $16x^{12} y^4$ .
2. Divide the numbers:  $16 \div 8 = 2$ .
3. Subtract powers:  $x^{12-5} = x^7$ ,  $y^{4-2} = y^2$ .

**$2x^7 y^2$**

## 4. Equations with the unknown in the power

☐ I revised this

- Write both sides as powers of the **same base**, then set the powers equal.
- Useful bases:  $4 = 2^2$ ,  $8 = 2^3$ ,  $16 = 2^4$ ,  $32 = 2^5$ ;  $9 = 3^2$ ,  $27 = 3^3$ ,  $81 = 3^4$ ;  $25 = 5^2$ ,  $125 = 5^3$ .
- $1/8 = 2^{-3}$  and  $\sqrt{2} = 2^{1/2}$  — negative and fractional powers often appear here.
- No logarithms are needed (or expected) at this level.

### How to do it

1. Choose a common base (usually 2, 3 or 5).
2. Rewrite each side as  $\text{base}^{\text{power}}$ , using  $(a^m)^n = a^{mn}$ .
3. Set the powers equal and solve the linear equation.

### Worked example

Solve  $4^{x+1} = 8^x$ .

1. Base 2:  $(2^2)^{x+1} = (2^3)^x$ .
2.  $2^{2x+2} = 2^{3x}$ .
3.  $2x + 2 = 3x$ .

**$x = 2$**

**Exam tip:** If one side is a fraction like  $1/27$ , write it as a negative power first:  $3^{-3}$ .

## Common mistakes

| Don't write                                            | Write instead                                            |
|--------------------------------------------------------|----------------------------------------------------------|
| $\times 2^3 \times 2^4 = 2^{12}$                       | $\checkmark$ Add the powers: $2^7$                       |
| $\times 2^3 \times 2^4 = 4^7$                          | $\checkmark$ The base stays 2: $2^7$                     |
| $\times 4^{-2} = -16$                                  | $\checkmark 4^{-2} = 1/16$                               |
| $\times (3 \times 2)^3 = 3 \times 6$                   | $\checkmark$ Cube the 3 too: $27 \times 8$               |
| $\times 5^0 = 0$                                       | $\checkmark 5^0 = 1$                                     |
| $\times 8^{2/3} = (8^2)^{1/3}$ worked out the long way | $\checkmark$ Root first is easier: $(\sqrt[3]{8})^2 = 4$ |

## In the exam

- Non-calculator questions on indices are common — practise without a calculator.
- For 'find n' questions, write every number as a power of the same base before comparing.
- Check by substituting your answer back in.

## My notes

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