

Number Sequences

Quick revision notes [Open it online](#)

Name: _____

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At a glance

Term-to-term rules, nth terms of linear, special, quadratic, cubic and exponential sequences, and shape patterns.

1. Term-to-term rules
2. Linear sequences: the nth term
3. Special sequences
4. Quadratic sequences
5. Cubic and exponential sequences
6. Patterns of shapes

1. Term-to-term rules

☐ I revised this

- The term-to-term rule says how to get from one term to the next: 3, 7, 11, 15 → add 4.
- Arithmetic (linear) sequences add the same each time; geometric ones multiply by the same each time.
- Look at the differences: constant first difference → linear; constant second difference → quadratic.

Worked example

Find the next two terms of 2, 6, 18, 54, ...

1. Each term $\times 3$.

162, 486

2. Linear sequences: the nth term

☐ I revised this

- nth term = $dn + (\text{first term} - d)$, where d is the common difference.
- To test whether a number is a term, set the nth term equal to it: a whole-number n means yes.
- To find which term a value is, solve nth term = value.

Formulas

nth term = $dn + (a - d)$

a = first term, d = difference

Worked example

Find the nth term of 7, 11, 15, 19, ... Is 101 a term?

1. $d = 4$: $4n$. $4n$ gives 4, 8, 12...; add 3.
2. nth term $4n + 3$.
3. $4n + 3 = 101 \rightarrow n = 24.5$, not a whole number.

$4n + 3$; 101 is not a term

3. Special sequences

☐ I revised this

- Recognise these and their shifts (+ or -) and multiples:

Sequence	Terms	nth term
Square	1, 4, 9, 16, 25	n^2
Cube	1, 8, 27, 64, 125	n^3
Triangle	1, 3, 6, 10, 15	$n(n + 1)/2$
Powers of 2	2, 4, 8, 16, 32	2^n
Square + 1	2, 5, 10, 17, 26	$n^2 + 1$
Twice square	2, 8, 18, 32, 50	$2n^2$

Worked example

Find the nth term of 0, 3, 8, 15, 24, ...

1. Compare with squares 1, 4, 9, 16, 25: each is 1 less.

$$n^2 - 1$$

4. Quadratic sequences

☐ I revised this

- If the **second** difference is constant, the nth term is $an^2 + bn + c$.
- $a = \text{second difference} \div 2$.

How to do it

1. Find the first and second differences.
2. $a = \text{second difference} \div 2$.
3. Subtract an^2 from each term.
4. Find the nth term of what is left (linear) and add it on.

Worked example

Find the nth term of 4, 10, 18, 28, 40, ...

1. Differences 6, 8, 10, 12; second difference 2, so $a = 1$.
2. Subtract n^2 (1, 4, 9, 16, 25): 3, 6, 9, 12, 15 = $3n$.

$$n^2 + 3n$$

5. Cubic and exponential sequences

☐ I revised this

- Constant **third** difference → cubic; often a simple form like $n^3 + 1$.
- A constant **ratio** → exponential: nth term = $a \times r^{n-1}$ ($a = \text{first term}$, $r = \text{ratio}$).
- Combinations: compare with the basic sequence and spot the change.

Formulas

$$\text{nth term} = a \times r^{n-1}$$

geometric (exponential)

Worked example

Find the nth term of 5, 10, 20, 40, ...

1. Ratio $r = 2$, first term 5.

$$5 \times 2^{n-1}$$

6. Patterns of shapes

☐ I revised this

- Count the items in the first few patterns and write them as a sequence.
- Find the n th term, then use it to predict any pattern or to find which pattern uses a given number.

Worked example

Pattern n uses $3n + 1$ matchsticks. Which pattern uses 100 matchsticks?

1. $3n + 1 = 100$.

2. $3n = 99$.

Pattern 33

Common mistakes

Don't write	Write instead
✗ n th term of 5, 8, 11 is $n + 3$	✓ The difference is 3, so $3n + 2$
✗ Using the second difference as a	✓ a is HALF the second difference
✗ Saying 101 is the 24.5th term	✓ n must be a whole number, so 101 is not a term
✗ 5×2^n for 5, 10, 20, 40	✓ Check $n = 1$: $5 \times 2^{n-1}$ gives 5

In the exam

- Always check your n th term with $n = 1$ and $n = 2$.
- Show the differences — they earn method marks.
- Give the n th term in simplest form ($3n + 2$, not $3n + 5 - 3$).

My notes

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