Proportionality

Quick revision notes

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Name: _____

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At a glance

Direct and inverse proportion with ∝ and k, including squares, cubes and roots.

1. Direct proportion
2. Inverse proportion
3. The effect of a change

1. Direct proportion

☐ I revised this

- $y \propto x$ means $y = kx$: when x doubles, y doubles. The graph is a straight line through the origin.
- Other direct forms: $y \propto x^2$ ($y = kx^2$), $y \propto x^3$, $y \propto \sqrt{x}$, $y \propto \sqrt[3]{x}$.
- Find k from a given pair of values, then use the formula.

Method for every proportion question

1. Write the formula with k.
2. Substitute the pair you are given to find k.
3. Write the formula again with the number for k.
4. Substitute the new value.

Formulas

$y \propto x \Rightarrow y = kx$
$y \propto x^2 \Rightarrow y = kx^2$
$y \propto \sqrt{x} \Rightarrow y = k\sqrt{x}$

Worked example

y is directly proportional to the square of x. y = 18 when x = 3. Find y when x = 5.

1. $y = kx^2$.
2. $18 = k \times 9$, so $k = 2$.
3. $y = 2x^2$.
4. $y = 2 \times 25$.

y = 50

2. Inverse proportion

☐ I revised this

- $y \propto 1/x$ means $y = k/x$: when x doubles, y halves. xy is always the same.
- Other inverse forms: $y \propto 1/x^2$, $y \propto 1/\sqrt{x}$.
- The graph of $y = k/x$ is a curve that gets close to both axes.

Formulas

$y \propto 1/x \Rightarrow y = k/x$
$y \propto 1/x^2 \Rightarrow y = k/x^2$

Worked example

The force F is inversely proportional to the square of the distance d. F = 20 when d = 3. Find F when d = 6.

1. $F = k/d^2$.
2. $20 = k/9$, so $k = 180$.
3. $F = 180/36$.

F = 5

3. The effect of a change

I revised this

- If $y \propto x^n$ and x is multiplied by m, then y is multiplied by m^n.
- Inverse: if $y \propto 1/x^n$ and x is multiplied by m, y is divided by m^n.
- Percentage changes work the same way: x up 10% means x × 1.1.

Relationship	x doubled → y	x tripled → y
$y \propto x$	× 2	× 3
$y \propto x^2$	× 4	× 9
$y \propto x^3$	× 8	× 27
$y \propto 1/x$	÷ 2	÷ 3
$y \propto 1/x^2$	÷ 4	÷ 9

Worked example

$y \propto x^3$. x is increased by 10%. By what percentage does y increase?

1. x is multiplied by 1.1.
2. y is multiplied by $1.1^3 = 1.331$.

33.1%

Common mistakes

Don't write	Write instead
✗ Using y = kx for 'proportional to the square'	✓ Square it: $y = kx^2$
✗ Inverse proportion written y = x/k	✓ It is $y = k/x$
✗ Not squaring when substituting into $y = kx^2$	✓ $18 = k \times 3^2$, not $18 = k \times 3$
✗ Forgetting to write the formula with k	✓ Write $y = 2x^2$ — it is often worth a mark

In the exam

- Read the wording carefully: 'square', 'square root', 'cube', 'inversely'.
- Always write the formula with your value of k in it before the final step.
- Check: a larger x should make y bigger (direct) or smaller (inverse).

My notes