

# Surds

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Name: \_\_\_\_\_

Date: \_\_\_\_\_

## At a glance

Exact roots: simplifying, calculating with surds, and rationalising denominators.

1. Simplifying surds
2. Adding, multiplying and expanding
3. Rationalising the denominator

## 1. Simplifying surds

☐ I revised this

- A **surd** is a root that is irrational, like  $\sqrt{2}$  or  $\sqrt{12}$ . It is exact — keep it as a surd unless told to round.
- Find the **largest square factor**:  $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$ .
- Square factors to look for: 4, 9, 16, 25, 36, 49, 64, 81, 100.

### Formulas

$$\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$$

$$\sqrt{a} \times \sqrt{a} = a$$

### Worked example

Simplify  $\sqrt{75} + \sqrt{12}$ .

1.  $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$ .
2.  $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ .
3.  $5\sqrt{3} + 2\sqrt{3}$ .

**$7\sqrt{3}$**

## 2. Adding, multiplying and expanding

☐ I revised this

- Only **like surds** add or subtract:  $6\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$ , but  $\sqrt{2} + \sqrt{3}$  cannot be combined.
- Multiply:  $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$ ;  $2\sqrt{5} \times 3\sqrt{2} = 6\sqrt{10}$ .
- Expand brackets just as in algebra, then simplify each term.
- $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$ : the surds cancel. This is used to rationalise.

### Worked example

Expand and simplify  $(3 + \sqrt{2})^2$ .

1.  $(3 + \sqrt{2})(3 + \sqrt{2}) = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$ .
2. Collect:  $11 + 6\sqrt{2}$ .

**$11 + 6\sqrt{2}$**

**Exam tip:**  $(a + \sqrt{b})^2$  is NOT  $a^2 + b$  — there is a middle term  $2a\sqrt{b}$ .

## 3. Rationalising the denominator

☐ I revised this

- 'Rationalise' means: no surd on the bottom of the fraction.
- $a/\sqrt{b}$ : multiply top and bottom by  $\sqrt{b}$ .  $10/\sqrt{5} = 10\sqrt{5}/5 = 2\sqrt{5}$ .
- $a/(b + \sqrt{c})$ : multiply top and bottom by  $(b - \sqrt{c})$ .  $a/(b - \sqrt{c})$ : multiply by  $(b + \sqrt{c})$ .

### How to do it

1. Look at the bottom.
2. Single surd  $\sqrt{b} \rightarrow$  multiply top and bottom by  $\sqrt{b}$ .
3.  $b \pm \sqrt{c} \rightarrow$  multiply top and bottom by the same with the opposite sign.
4. Expand and simplify; cancel any common factor.

### Worked example

Rationalise  $6/(3 - \sqrt{3})$ .

1. Multiply by  $(3 + \sqrt{3})/(3 + \sqrt{3})$ .
2. Bottom:  $9 - 3 = 6$ .
3. Top:  $6(3 + \sqrt{3})$ .
4.  $6(3 + \sqrt{3})/6$ .

**$3 + \sqrt{3}$**

### Common mistakes

| Don't write                                          | Write instead                                                  |
|------------------------------------------------------|----------------------------------------------------------------|
| $\times \sqrt{9} + \sqrt{16} = \sqrt{25}$            | $\checkmark \sqrt{9} + \sqrt{16} = 3 + 4 = 7$                  |
| $\times$ Leaving $\sqrt{72}$ as $2\sqrt{18}$         | $\checkmark$ Use the largest square factor: $6\sqrt{2}$        |
| $\times (3 + \sqrt{2})^2 = 9 + 2$                    | $\checkmark$ Middle term: $9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$ |
| $\times$ Multiplying only the top when rationalising | $\checkmark$ Multiply top and bottom by the same surd          |

### In the exam

- 'Give your answer in the form  $a + b\sqrt{c}$ ' — make sure  $a, b, c$  are integers and the surd is simplified.
- Surd questions are on the non-calculator paper; check by squaring your answer mentally.
- Exact answers (like in Pythagoras) are often wanted as simplified surds.

### My notes

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