

Functions

Quick revision notes [Open it online](#)

Name: _____

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At a glance

Function notation, inverses, composite functions, domain, range and asymptotes.

1. Function notation
2. Inverse functions
3. Composite functions
4. Domain, range and asymptotes

1. Function notation

☐ I revised this

- $f(x) = 3x - 4$ is a rule: $f(2)$ means put 2 in place of x : $f(2) = 2$.
- Solve $f(x) = 11$ by writing $3x - 4 = 11$.
- $f(x + 1)$ means replace x by $(x + 1)$: $3(x + 1) - 4 = 3x - 1$.
- $2f(x) = 2(3x - 4) = 6x - 8$ but $f(2x) = 6x - 4$.

Worked example

$g(x) = x^2 - 2x$. Find $g(-3)$ and solve $g(x) = 8$.

1. $g(-3) = 9 + 6 = 15$.
2. $x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0$.

$g(-3) = 15$; $x = 4$ or $x = -2$

2. Inverse functions

☐ I revised this

- f^{-1} undoes f : if $f(3) = 11$ then $f^{-1}(11) = 3$.
- $f^{-1}(x)$ is not $1/f(x)$.

How to do it

1. Write $y = f(x)$.
2. Swap x and y .
3. Make y the subject.
4. Write $f^{-1}(x) = \dots$

Worked example

$f(x) = (2x - 5)/3$. Find $f^{-1}(x)$.

1. $y = (2x - 5)/3$; swap: $x = (2y - 5)/3$.
2. $3x = 2y - 5$.
3. $2y = 3x + 5$.

$f^{-1}(x) = (3x + 5)/2$

Exam tip: Check: $f^{-1}(f(x))$ must give x back. Try a number.

3. Composite functions

☐ I revised this

- $fg(x)$ means $f(g(x))$: **do g first**, then f .
- Replace the x in f with the whole expression $g(x)$, in brackets.
- $ff(x)$ means apply f twice. fg and gf are usually different.

Worked example

$f(x) = 2x + 1$ and $g(x) = x^2$. Find $fg(x)$, $gf(x)$ and $fg(3)$.

1. $fg(x) = f(x^2) = 2x^2 + 1$.

2. $gf(x) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$.

3. $fg(3) = 2 \times 9 + 1 = 19$.

$fg(x) = 2x^2 + 1$, $gf(x) = 4x^2 + 4x + 1$, $fg(3) = 19$

4. Domain, range and asymptotes

☐ I revised this

- **Domain:** the x values allowed. Leave out any value that makes a denominator zero.
- **Range:** the y values you get out. x^2 is never negative, so the range of $f(x) = x^2$ is $f(x) \geq 0$.
- $y = k/(x - a) + b$ has asymptotes $x = a$ (vertical) and $y = b$ (horizontal).
- $y = a^x + b$ has the horizontal asymptote $y = b$.

Worked example

$f(x) = 3/(x - 2) + 1$. State the value excluded from the domain and the asymptotes.

1. $x - 2 = 0$ when $x = 2$.

2. As x gets large, $3/(x - 2) \rightarrow 0$, so $y \rightarrow 1$.

$x = 2$ excluded; asymptotes $x = 2$ and $y = 1$

Common mistakes

Don't write	Write instead
$\times$ $fg(2)$: working out f first	$\checkmark$ fg means $f(g(2))$: do g first
$\times$ $f^{-1}(x) = 1/f(x)$	$\checkmark$ The inverse undoes f ; it is not the reciprocal
$\times$ $gf(x) = 2x + 1^2$	$\checkmark$ Use brackets: $(2x + 1)^2$
$\times$ $2f(x)$ and $f(2x)$ treated as the same	$\checkmark$ $2f(x)$ doubles the output; $f(2x)$ doubles the input

In the exam

- Use brackets every time you substitute an expression.
- Check an inverse with a number: $f(2)$ then f^{-1} of the answer should give 2.
- For 'solve $ff(x) = \dots$ ' find $ff(x)$ as an expression first.

My notes