

Mathematics 0580

Quick revision notes — every chapter

Name: _____

Class: _____

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Arithmetics

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Name: _____

Date: _____

At a glance

The number toolkit used in every other chapter: kinds of number, BIDMAS, fractions, decimals and recurring decimals.

1. Types of number
2. Order of operations and negative numbers
3. Fractions
4. Decimals and ordering
5. Halfway values and differences
6. Recurring decimals to fractions

1. Types of number

☐ I revised this

- **Natural numbers:** 1, 2, 3, ... **Integers:** whole numbers including 0 and negatives (... , -2, -1, 0, 1, 2, ...).
- **Prime:** exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... 1 is **not** prime; 2 is the only even prime.
- **Factor** of 12: divides into it (1, 2, 3, 4, 6, 12). **Multiple** of 12: in its times table (12, 24, 36, ...).
- **Rational:** can be written as a fraction a/b (all integers, terminating and recurring decimals). **Irrational:** cannot — π , $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and the square root of any non-square number.
- **Reciprocal** of x is $1/x$: reciprocal of 4 is $1/4$, of $2/5$ is $5/2$. A number times its reciprocal is 1.

n	1	2	3	4	5	6	7	8	9	10
n^2	1	4	9	16	25	36	49	64	81	100
n^3	1	8	27	64	125	216	343	512	729	1000

Worked example

From -3, 0, $1/2$, $\sqrt{9}$, $\sqrt{5}$, 7 and 0.333..., list (a) the irrational numbers, (b) the prime numbers.

1. $\sqrt{9} = 3$ and $0.333... = 1/3$, so both are rational.
2. $\sqrt{5}$ is not a whole number and does not recur: irrational.
3. Primes are positive whole numbers with exactly two factors: $\sqrt{9} = 3$ and 7.

(a) $\sqrt{5}$ (b) $\sqrt{9}$ and 7

Exam tip: Also know $11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$ for the non-calculator paper.

2. Order of operations and negative numbers

☐ I revised this

- **BIDMAS:** Brackets → Indices → Division and Multiplication (left to right) → Addition and Subtraction (left to right).
- Two signs next to each other: same signs give +, different signs give -. So $5 - (-3) = 8$ and $4 \times (-2) = -8$.
- $(-3)^2 = 9$ but $-3^2 = -9$: without brackets only the 3 is squared.
- A fraction line acts like brackets: $(8 + 4) \div (5 - 2) = 4$. On a calculator, put brackets round the top and the bottom.

Order to work in

1. Work out anything in brackets.
2. Then powers and roots.
3. Then $\times$ and $\div$ from left to right.
4. Finally + and - from left to right.

Worked example

Work out $(-3)^2 - 4 \times (5 - 7)$.

1. Brackets: $5 - 7 = -2$.
2. Power: $(-3)^2 = 9$.
3. Multiply: $4 \times (-2) = -8$.
4. $9 - (-8) = 9 + 8$.

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3. Fractions

☐ I revised this

- To add or subtract, use a **common denominator** (the LCM of the bottoms), then add the tops.
- To multiply, multiply tops and bottoms (cancel first to keep numbers small).
- To divide, **flip the second fraction and multiply**: $a/b \div c/d = a/b \times d/c$.
- Change mixed numbers to improper fractions before multiplying or dividing: $2\frac{3}{4} = 11/4$.
- Fraction of an amount: divide by the bottom, multiply by the top. $3/8$ of 240 = $240 \div 8 \times 3 = 90$.
- Always give the answer in its simplest form.

Formulas

$a/b + c/d = (ad + bc)/bd$	then simplify
$a/b \times c/d = ac/bd$	multiply across
$a/b \div c/d = a/b \times d/c$	flip the second, then multiply

Worked example

Work out $2\frac{3}{4} - 1\frac{5}{6}$.

1. Improper fractions: $11/4 - 11/6$.
2. Common denominator 12: $33/12 - 22/12$.
3. $= 11/12$.

11/12

Exam tip: On the non-calculator paper write every step: the method marks need the common denominator to be seen.

4. Decimals and ordering

☐ I revised this

- Multiplying decimals: ignore the points, multiply, then give the answer as many decimal places as the question had in total. $0.3 \times 0.04 = 0.012$.
- Dividing by a decimal: multiply both numbers by 10, 100, ... until you divide by a whole number. $4.8 \div 0.06 = 480 \div 6 = 80$.
- Fraction $\rightarrow$ decimal: divide the top by the bottom. $3/8 = 0.375$.
- To order a mixture of fractions, decimals and percentages, change them all to decimals (to 3 or 4 places).
- Symbols: = equal, $\neq$ not equal, > greater than, < less than, $\geq$ greater than or equal to, $\leq$ less than or equal to.

Worked example

Write in order, smallest first: 0.7, $2/3$, 68%, 0.67.

1. $2/3 = 0.6667$, $68\% = 0.68$.
2. Compare: 0.6667, 0.67, 0.68, 0.7.

2/3, 0.67, 68%, 0.7

5. Halfway values and differences

☐ I revised this

- The number **halfway** between a and b is $(a + b) \div 2$.
- The **difference** between two numbers is the larger minus the smaller; across zero, add the distances: from -12 to 9 is $12 + 9 = 21$.
- Temperature questions: a rise from -7°C to 5°C is 12 degrees; a fall of 9 degrees from 4°C gives -5°C .

Formulas

$$\text{halfway} = (a + b) \div 2$$

works with negatives and decimals

Worked example

Find the number halfway between -2.6 and 4.2 .

1. Add: $-2.6 + 4.2 = 1.6$.

2. Halve: $1.6 \div 2$.

0.8

6. Recurring decimals to fractions

☐ I revised this

- A dot over a digit means it repeats: $0.4' = 0.444\dots$, $0.27' = 0.272727\dots$
- Every recurring decimal is a fraction. One repeating digit $\rightarrow$ over 9; two repeating digits $\rightarrow$ over 99.
- If some digits do not repeat ($0.3666\dots$), multiply twice so the repeating parts line up, then subtract.

How to do it

1. Let x = the decimal.
2. Multiply by 10 (one repeating digit) or 100 (two) so the repeating part lines up.
3. Subtract to remove the repeating part.
4. Divide and simplify.

Fraction	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{1}{11}$	$\frac{1}{6}$
Decimal	$0.3'$	$0.6'$	$0.1'$	$0.09'$	$0.16'$

Worked example

Write $0.3666\dots$ as a fraction in its simplest form.

1. $x = 0.3666\dots$, $10x = 3.666\dots$, $100x = 36.666\dots$

2. $100x - 10x: 90x = 33$.

3. $x = 33/90$.

$\frac{11}{30}$

Exam tip: 'Show that $0.27 = 3/11$ ' needs the algebra ($x = \dots$, $100x = \dots$, subtract). Just dividing 3 by 11 gets no marks.

Common mistakes

Don't write	Write instead
✗ Calling 1 a prime number	✓ 1 has only one factor, so it is not prime
✗ $3 + 4 \times 2 = 14$	✓ Multiply first: $3 + 8 = 11$
✗ $-3^2 = 9$	✓ Only the 3 is squared: -9 . $(-3)^2 = 9$
✗ $1/2 + 1/3 = 2/5$	✓ Common denominator: $3/6 + 2/6 = 5/6$
✗ $2^{3/4} \times 2 = 4^{3/4}$	✓ Improper fraction first: $11/4 \times 2 = 11/2 = 5\frac{1}{2}$
✗ Ordering 0.7 before 0.67 because $7 < 67$	✓ Compare place by place: $0.70 > 0.67$

In the exam

- Non-calculator questions want every step shown — a correct answer with no working may lose marks.
- Give fractions in their simplest form unless the question asks otherwise.
- Check an answer is sensible: dividing by a number less than 1 makes it bigger.

My notes

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Indices

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Name: _____

Date: _____

At a glance

Powers and roots: the four laws, zero/negative/fractional powers, algebra with powers, and index equations.

1. The laws of indices
2. Zero, negative and fractional powers
3. Indices in algebra
4. Equations with the unknown in the power

1. The laws of indices

☐ I revised this

- The **index** (power) says how many times a number is multiplied by itself: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.
- The laws only work when the **base is the same**: $2^3 \times 2^4 = 2^7$, but $2^3 \times 3^4$ cannot be combined.
- Numbers in front are dealt with normally: $3x^2 \times 4x^5 = 12x^7$.

Formulas

$a^m \times a^n = a^{m+n}$	multiply $\rightarrow$ add the powers
$a^m \div a^n = a^{m-n}$	divide $\rightarrow$ subtract the powers
$(a^m)^n = a^{mn}$	power of a power $\rightarrow$ multiply
$(ab)^n = a^n b^n$	everything inside is raised

Worked example

Simplify $2^5 \times 2^3 \div 2^4$ and find its value.

1. Add: $5 + 3 = 8$.
2. Subtract: $8 - 4 = 4$.
3. $2^4 = 16$.

$2^4 = 16$

2. Zero, negative and fractional powers

☐ I revised this

- $a^0 = 1$ for any a except 0: $7^0 = 1$, $(-5)^0 = 1$.
- A **negative** power means 'one over': $a^{-n} = 1/a^n$. It never makes the answer negative: $2^{-3} = 1/8$.
- A fraction raised to a negative power flips: $(2/3)^{-2} = (3/2)^2 = 9/4$.
- A **fractional** power is a root: $a^{1/2} = \sqrt{a}$, $a^{1/3} = \sqrt[3]{a}$, and $a^{m/n} = (\sqrt[n]{a})^m$ — do the root first.
- Negative fractional: do all three — flip, root, power. $8^{-2/3} = 1/(\sqrt[3]{8})^2 = 1/4$.

For $a^{-m/n}$

1. Negative sign? Flip the number (write 1 over it).
2. Bottom of the fraction? Take that root.
3. Top of the fraction? Raise to that power.

Formulas

$a^0 = 1$	
$a^{-n} = 1/a^n$	one over
$a^{1/n} = \sqrt[n]{a}$	root
$a^{m/n} = (\sqrt[n]{a})^m$	root first, then power

Worked example

Find the value of $(27/8)^{-2/3}$.

1. Flip: $(8/27)^{2/3}$.
2. Cube root: $\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$, giving $2/3$.
3. Square: $(2/3)^2$.

$4/9$

Exam tip: Know the powers of 2 (2, 4, 8, 16, 32, 64, 128), 3 (3, 9, 27, 81, 243) and 5 (5, 25, 125, 625) by heart.

3. Indices in algebra

☐ I revised this

- Deal with the numbers and each letter separately.
- $(3x^2)^3 = 3^3 \times x^6 = 27x^6$ — the number in front is cubed too.
- $12a^5 \div 3a^{-2} = 4a^{5-(-2)} = 4a^7$.
- $(2x^{3/5})^2 = 4x^{6/25}$: square the top and the bottom.

Worked example

Simplify $(2x^3 y)^4 \div 8x^5 y^2$.

1. Expand the bracket: $16x^{12} y^4$.
2. Divide the numbers: $16 \div 8 = 2$.
3. Subtract powers: $x^{12-5} = x^7$, $y^{4-2} = y^2$.

$2x^7 y^2$

4. Equations with the unknown in the power

☐ I revised this

- Write both sides as powers of the **same base**, then set the powers equal.
- Useful bases: $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, $32 = 2^5$; $9 = 3^2$, $27 = 3^3$, $81 = 3^4$; $25 = 5^2$, $125 = 5^3$.
- $1/8 = 2^{-3}$ and $\sqrt{2} = 2^{1/2}$ — negative and fractional powers often appear here.
- No logarithms are needed (or expected) at this level.

How to do it

1. Choose a common base (usually 2, 3 or 5).
2. Rewrite each side as $\text{base}^{\text{power}}$, using $(a^m)^n = a^{mn}$.
3. Set the powers equal and solve the linear equation.

Worked example

Solve $4^{x+1} = 8^x$.

1. Base 2: $(2^2)^{x+1} = (2^3)^x$.
2. $2^{2x+2} = 2^{3x}$.
3. $2x + 2 = 3x$.

$x = 2$

Exam tip: If one side is a fraction like $1/27$, write it as a negative power first: 3^{-3} .

Common mistakes

Don't write	Write instead
$\times 2^3 \times 2^4 = 2^{12}$	$\checkmark$ Add the powers: 2^7
$\times 2^3 \times 2^4 = 4^7$	$\checkmark$ The base stays 2: 2^7
$\times 4^{-2} = -16$	$\checkmark 4^{-2} = 1/16$
$\times (3 \times 2)^3 = 3 \times 6$	$\checkmark$ Cube the 3 too: 27×6
$\times 5^0 = 0$	$\checkmark 5^0 = 1$
$\times 8^{2/3} = (8^2)^{1/3}$ worked out the long way	$\checkmark$ Root first is easier: $(\sqrt[3]{8})^2 = 4$

In the exam

- Non-calculator questions on indices are common — practise without a calculator.
- For 'find n' questions, write every number as a power of the same base before comparing.
- Check by substituting your answer back in.

My notes

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Standard Form

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Name: _____

Date: _____

At a glance

Writing very large and very small numbers as $A \times 10^n$, ordering them, and calculating with them.

1. What standard form is
2. Ordering numbers in standard form
3. Calculating without a calculator
4. Using a calculator and real problems

1. What standard form is

☐ I revised this

- A number in standard form is $A \times 10^n$ with $1 \leq A < 10$ and n a whole number.
- Large numbers have a positive power: $4\,560\,000 = 4.56 \times 10^6$.
- Numbers less than 1 have a negative power: $0.00072 = 7.2 \times 10^{-4}$.
- The power is how many places the decimal point moves to get A .

How to do it

1. Write the digits with the point after the first non-zero digit (that is A).
2. Count the places the point moved.
3. Moved left (big number) $\rightarrow$ positive power; moved right (small number) $\rightarrow$ negative power.

Ordinary	Standard form
300 000	3×10^5
45 600	4.56×10^4
0.05	5×10^{-2}
0.000 307	3.07×10^{-4}

Worked example

Write 0.000 307 in standard form.

1. First non-zero digit is 3: $A = 3.07$.
2. The point moves 4 places to the right.

3.07×10^{-4}

2. Ordering numbers in standard form

☐ I revised this

- Compare the powers first: the bigger power is the bigger number (for positive numbers).
- If the powers are equal, compare the A values.
- Negative powers: 10^{-2} is bigger than 10^{-3} .

Worked example

Order, smallest first: 3.1×10^{-2} , 9.8×10^{-3} , 1.2×10^{-2} .

1. 10^{-3} is the smallest power, so 9.8×10^{-3} is smallest.
2. Then 1.2×10^{-2} before 3.1×10^{-2} .

9.8×10^{-3} , 1.2×10^{-2} , 3.1×10^{-2}

3. Calculating without a calculator

☐ I revised this

- Multiply: multiply the A values and **add** the powers.
- Divide: divide the A values and **subtract** the powers.
- Then fix the answer if A is not between 1 and 10: $12 \times 10^9 = 1.2 \times 10^{10}$; $0.5 \times 10^3 = 5 \times 10^2$.
- Add or subtract: change both numbers to ordinary numbers (or to the same power), then add.

Formulas

$$(a \times 10^m)(b \times 10^n) = ab \times 10^{m+n}$$

$$(a \times 10^m) \div (b \times 10^n) = (a/b) \times 10^{m-n}$$

Worked example

Work out $(6 \times 10^5) \div (1.5 \times 10^{-2})$.

1. $6 \div 1.5 = 4$.

2. $10^5 - (-2) = 10^7$.

4×10^7

Exam tip: Adding: $5 \times 10^3 + 7 \times 10^2 = 5000 + 700 = 5700 = 5.7 \times 10^3$.

4. Using a calculator and real problems

☐ I revised this

- Use the $\times 10^x$ (EXP) key: type $3.2 \times 10^x 5$, not $3.2 \times 10 \wedge 5$ (which can go wrong with division).
- Give the final answer in standard form if the question uses it, rounded to 3 significant figures.
- Typical contexts: distances in space, masses of atoms, populations, speed of light (3×10^8 m/s).

Worked example

Light travels 3×10^8 m each second. How long does it take to travel 1.5×10^{11} m?

1. time = distance $\div$ speed.

2. $1.5 \div 3 = 0.5$ and $10^{11-8} = 10^3$.

3. $0.5 \times 10^3 = 500$.

500 seconds

Common mistakes

Don't write	Write instead
$\times$ Leaving 45.6×10^5 as the answer	$\checkmark$ A must be between 1 and 10: 4.56×10^6
$\times$ $0.00072 = 7.2 \times 10^4$	$\checkmark$ Small numbers need a negative power: 7.2×10^{-4}
$\times$ Adding the powers when adding numbers	$\checkmark$ Only multiplication adds the powers
$\times$ Calculator answer 2.4E7 copied as 2.4^7	$\checkmark$ It means 2.4×10^7

In the exam

- Read whether the answer must be in standard form — then give it in standard form.
- On the non-calculator paper, show the separate steps for the numbers and the powers.
- Check size: a speed or distance should be sensible after you divide.

My notes

Significant Figures

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Name: _____

Date: _____

At a glance

Decimal places, significant figures, estimating, and how accurate an answer should be.

1. Rounding to decimal places
2. Rounding to significant figures
3. Estimating a calculation
4. Choosing a sensible accuracy

1. Rounding to decimal places

☐ I revised this

- Count the places after the decimal point, then look at the **next** digit.
- 5 or more → round up; 4 or less → leave the digit.
- Keep the trailing zero if the question asks for it: 2.497 to 2 d.p. is 2.50, not 2.5.

Worked example

Round 3.14159 to 2 decimal places.

1. Two places: 3.14|159.
2. The next digit is 1 → stays.

3.14

2. Rounding to significant figures

☐ I revised this

- The **first significant figure** is the first non-zero digit.
- Zeros at the start of a decimal never count: 0.004682 → first s.f. is 4.
- Zeros between non-zero digits do count: 3005 has 4 s.f.
- Keep the size of the number with place-value zeros: 68 451 to 3 s.f. = 68 500.

Number	1 s.f.	2 s.f.	3 s.f.
68 451	70 000	68 000	68 500
0.004682	0.005	0.0047	0.00468
2.0496	2	2.0	2.05

Worked example

Round 0.004682 to 2 significant figures.

1. First s.f. is the 4; two s.f. = 0.0046|82.
2. Next digit 8 → round up.

0.0047

3. Estimating a calculation

☐ I revised this

- Round **every** number to 1 significant figure, then work it out without a calculator.
- Show the rounded numbers — the method mark is for them.
- Dividing by a decimal: $200 \div 0.2 = 2000 \div 2 = 1000$.
- Overestimate or underestimate? If you rounded the numbers you multiply up, the estimate is too big.

1. Round each number to 1 s.f.
2. Write the new calculation down.
3. Work it out (simplify fractions first).

Estimate $(38.7 \times 5.12) \div 0.198$.

1. Round: $40 \times 5 \div 0.2$.
2. $40 \times 5 = 200$.
3. $200 \div 0.2 = 1000$.

Exam tip: A square root estimate: round to the nearest square number, e.g. $\sqrt{48.6} \approx \sqrt{49} = 7$.

☐ I revised this

- Unless the question says otherwise, give answers to **3 significant figures** (or exact).
- Angles in degrees: 1 decimal place. Money: to the nearest cent.
- Never round in the middle of a calculation — keep the full calculator value and round only the final answer.
- If the answer is exact (like 7.25), write it in full rather than rounding it.

A rectangle is 3.47 m by 2.6 m. Find its area to a sensible accuracy.

1. $3.47 \times 2.6 = 9.022$.
2. Round to 3 s.f.

Common mistakes

Don't write	Write instead
✗ 68 451 to 3 s.f. = 685	✓ Keep the place-value zeros: 68 500
✗ 0.004682 to 2 s.f. = 0.00	✓ Start counting at the first non-zero digit: 0.0047
✗ Rounding halfway through a calculation	✓ Keep full values; round at the end
✗ Estimating with the exact numbers	✓ Round each number to 1 s.f. first and show it

- Write the full calculator answer down before you round it — it can earn a mark if your rounding slips.
- If the question says 'to 2 d.p.' give exactly 2 decimal places, including a final zero.
- An answer to 3 s.f. is the Cambridge default: $2.71828 \rightarrow 2.72$.

Functions

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Name: _____

Date: _____

At a glance

Function notation, inverses, composite functions, domain, range and asymptotes.

1. Function notation
2. Inverse functions
3. Composite functions
4. Domain, range and asymptotes

1. Function notation

☐ I revised this

- $f(x) = 3x - 4$ is a rule: $f(2)$ means put 2 in place of x : $f(2) = 2$.
- Solve $f(x) = 11$ by writing $3x - 4 = 11$.
- $f(x + 1)$ means replace x by $(x + 1)$: $3(x + 1) - 4 = 3x - 1$.
- $2f(x) = 2(3x - 4) = 6x - 8$ but $f(2x) = 6x - 4$.

Worked example

$g(x) = x^2 - 2x$. Find $g(-3)$ and solve $g(x) = 8$.

1. $g(-3) = 9 + 6 = 15$.
2. $x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0$.

$g(-3) = 15$; $x = 4$ or $x = -2$

2. Inverse functions

☐ I revised this

- f^{-1} undoes f : if $f(3) = 11$ then $f^{-1}(11) = 3$.
- $f^{-1}(x)$ is not $1/f(x)$.

How to do it

1. Write $y = f(x)$.
2. Swap x and y .
3. Make y the subject.
4. Write $f^{-1}(x) = \dots$

Worked example

$f(x) = (2x - 5)/3$. Find $f^{-1}(x)$.

1. $y = (2x - 5)/3$; swap: $x = (2y - 5)/3$.
2. $3x = 2y - 5$.
3. $2y = 3x + 5$.

$f^{-1}(x) = (3x + 5)/2$

Exam tip: Check: $f^{-1}(f(x))$ must give x back. Try a number.

3. Composite functions

☐ I revised this

- $fg(x)$ means $f(g(x))$: **do g first**, then f .
- Replace the x in f with the whole expression $g(x)$, in brackets.
- $ff(x)$ means apply f twice. fg and gf are usually different.

Worked example

$f(x) = 2x + 1$ and $g(x) = x^2$. Find $fg(x)$, $gf(x)$ and $fg(3)$.

1. $fg(x) = f(x^2) = 2x^2 + 1$.

2. $gf(x) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$.

3. $fg(3) = 2 \times 9 + 1 = 19$.

$fg(x) = 2x^2 + 1$, $gf(x) = 4x^2 + 4x + 1$, $fg(3) = 19$

4. Domain, range and asymptotes

☐ I revised this

- **Domain:** the x values allowed. Leave out any value that makes a denominator zero.
- **Range:** the y values you get out. x^2 is never negative, so the range of $f(x) = x^2$ is $f(x) \geq 0$.
- $y = k/(x - a) + b$ has asymptotes $x = a$ (vertical) and $y = b$ (horizontal).
- $y = a^x + b$ has the horizontal asymptote $y = b$.

Worked example

$f(x) = 3/(x - 2) + 1$. State the value excluded from the domain and the asymptotes.

1. $x - 2 = 0$ when $x = 2$.

2. As x gets large, $3/(x - 2) \rightarrow 0$, so $y \rightarrow 1$.

$x = 2$ excluded; asymptotes $x = 2$ and $y = 1$

Common mistakes

Don't write	Write instead
$\times$ $fg(2)$: working out f first	$\checkmark$ fg means $f(g(2))$: do g first
$\times$ $f^{-1}(x) = 1/f(x)$	$\checkmark$ The inverse undoes f ; it is not the reciprocal
$\times$ $gf(x) = 2x + 1^2$	$\checkmark$ Use brackets: $(2x + 1)^2$
$\times$ $2f(x)$ and $f(2x)$ treated as the same	$\checkmark$ $2f(x)$ doubles the output; $f(2x)$ doubles the input

In the exam

- Use brackets every time you substitute an expression.
- Check an inverse with a number: $f(2)$ then f^{-1} of the answer should give 2.
- For 'solve $ff(x) = \dots$ ' find $ff(x)$ as an expression first.

My notes

Factorization

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Common factors, difference of two squares, quadratics, grouping, and solving by factorising.

1. Common factors
2. Difference of two squares
3. Quadratics $x^2 + bx + c$
4. Quadratics $ax^2 + bx + c$
5. Four terms: grouping
6. Solving by factorising

1. Common factors

☐ I revised this

- Take out the **highest common factor** of every term — numbers and letters.
- 'Factorise fully' means nothing more can be taken out.
- Check by expanding back.

Worked example

Factorise fully $12x^2y - 18xy^2$.

1. HCF of 12 and 18 is 6; of x^2y and xy^2 is xy .
2. $12x^2y \div 6xy = 2x$ and $18xy^2 \div 6xy = 3y$.

$6xy(2x - 3y)$

2. Difference of two squares

☐ I revised this

- $a^2 - b^2 = (a - b)(a + b)$. Both terms must be squares and there must be a minus between them.
- $16x^2 - 25y^2 = (4x - 5y)(4x + 5y)$.
- Take out a common factor first: $2x^2 - 50 = 2(x^2 - 25) = 2(x - 5)(x + 5)$.
- $a^2 + b^2$ does not factorise.

Formulas

$$a^2 - b^2 = (a - b)(a + b)$$

Worked example

Factorise fully $3x^2 - 75$.

1. Common factor 3: $3(x^2 - 25)$.
2. $x^2 - 25 = (x - 5)(x + 5)$.

$3(x - 5)(x + 5)$

3. Quadratics $x^2 + bx + c$

☐ I revised this

- Find two numbers that **multiply to c** and **add to b**.
- If c is positive, both numbers have the sign of b. If c is negative, the signs are different.

Expression	Numbers	Factorised
$x^2 + 7x + 12$	3, 4	$(x + 3)(x + 4)$
$x^2 - 7x + 12$	-3, -4	$(x - 3)(x - 4)$
$x^2 + x - 12$	4, -3	$(x + 4)(x - 3)$
$x^2 - x - 12$	-4, 3	$(x - 4)(x + 3)$

Worked example

Factorise $x^2 + 2x - 15$.

1. Multiply to -15, add to 2: 5 and -3.

$$(x + 5)(x - 3)$$

4. Quadratics $ax^2 + bx + c$

☐ I revised this

- Use the **ac method**: find two numbers that multiply to $a \times c$ and add to b .
- Split the middle term with them, then factorise in pairs.

ac method

1. Multiply $a \times c$.
2. Find two numbers with that product and sum b .
3. Rewrite bx as the two terms.
4. Factorise the first pair and the second pair.
5. Take out the common bracket.

Worked example

Factorise $6x^2 + x - 2$.

1. $ac = -12$; numbers 4 and -3.
2. $6x^2 + 4x - 3x - 2$.
3. $2x(3x + 2) - 1(3x + 2)$.

$$(2x - 1)(3x + 2)$$

5. Four terms: grouping

☐ I revised this

- Group into two pairs, factorise each pair, then take out the common bracket.
- If the brackets don't match, try grouping the terms differently.

Worked example

Factorise $xy - 3x + 2y - 6$.

1. $x(y - 3) + 2(y - 3)$.

$$(x + 2)(y - 3)$$

6. Solving by factorising

☐ I revised this

- Rearrange to $= 0$, factorise, then set each factor equal to 0.
- $x(x - 5) = 0$ gives $x = 0$ or $x = 5$ — never divide both sides by x .
- $(2x + 3)(x - 3) = 0$ gives $x = -3/2$ or $x = 3$.

Worked example

Solve $2x^2 - 3x - 9 = 0$.

1. $ac = -18$: -6 and 3 .

2. $2x^2 - 6x + 3x - 9 = 2x(x - 3) + 3(x - 3)$.

3. $(2x + 3)(x - 3) = 0$.

$x = 3$ or $x = -1.5$

Common mistakes

Don't write	Write instead
$\times$ $16x^2 - 25 = (4x - 5)^2$	$\checkmark$ That expands to $16x^2 - 40x + 25$. Use $(4x - 5)(4x + 5)$
$\times$ $6x^2 + 9x = 3(2x^2 + 3x)$	$\checkmark$ Take out $3x$ as well: $3x(2x + 3)$
$\times$ $(x + 5)(x - 3) = 0 \rightarrow x = 5$ or -3	$\checkmark$ Change the signs: $x = -5$ or 3
$\times$ Dividing $x^2 = 5x$ by x	$\checkmark$ Rearrange: $x(x - 5) = 0$, so $x = 0$ or 5

In the exam

- Always check a factorisation by expanding it in your head.
- If a quadratic will not factorise, the question probably wants the formula.
- 'Hence solve' means use the factorisation you have just found.

My notes

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Statistics

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Averages and spread, grouped data, charts, histograms, cumulative frequency and scatter diagrams.

1. Averages and range
2. Grouped data
3. Charts and diagrams
4. Histograms
5. Cumulative frequency, quartiles and IQR
6. Scatter diagrams

1. Averages and range

☐ I revised this

- **Mean** = total $\div$ number of values. **Median** = middle value in order (average the middle two if there is an even count).
- **Mode** = most common value. **Range** = largest – smallest (a measure of spread).
- From a frequency table: mean = $\Sigma fx \div \Sigma f$; median position = $(n + 1) \div 2$.
- Missing value from a mean: total = mean $\times$ number of values.

Formulas

mean = $\Sigma fx \div \Sigma f$	frequency table
median position = $(n + 1) \div 2$	

Worked example

Scores 1, 2 and 3 have frequencies 4, 5 and 1. Find the mean and the median.

1. $\Sigma fx = 4 + 10 + 3 = 17$; $\Sigma f = 10$.
2. Mean = 1.7.
3. Median: position 5.5 — the 5th and 6th values are both 2.

Mean 1.7, median 2

2. Grouped data

☐ I revised this

- You don't know the exact values, so use each class's **midpoint**: the mean is an **estimate**.
- The **modal class** is the class with the highest frequency (for unequal widths, the highest frequency density).
- The class containing the median: find the $(n/2)$ th value by adding up frequencies.

Worked example

Heights 150–160 cm ($f = 4$), 160–170 ($f = 10$), 170–180 ($f = 6$). Estimate the mean.

1. Midpoints 155, 165, 175.
2. $\Sigma fx = 620 + 1650 + 1050 = 3320$; $\Sigma f = 20$.

Estimated mean 166 cm

3. Charts and diagrams

☐ I revised this

- Bar charts (gaps, equal widths), pictograms (with a key), pie charts (angle = frequency $\div$ total $\times 360^\circ$).
- Stem-and-leaf: always give a key ($3 \mid 4 = 34$); leaves in order; read the median by counting.
- Compare two distributions with an average AND a measure of spread, in context.

Worked example

A stem-and-leaf diagram has key $3 \mid 4 = 34$. What does $5 \mid 1$ mean?

1. Stem = tens, leaf = units.

51

4. Histograms

☐ I revised this

- With unequal class widths the vertical axis is **frequency density** = frequency $\div$ class width.
- The **area** of each bar is the frequency.
- To read a frequency: frequency density $\times$ class width. For part of a class, take that fraction of the bar.

Formulas

frequency density = frequency $\div$ class width

frequency = density $\times$ width

Worked example

A bar for $10 < x \leq 25$ has frequency density 2. A bar for $25 < x \leq 30$ has frequency 15. Find the first frequency and the second density.

1. Width 15: $2 \times 15 = 30$.
2. Width 5: $15 \div 5 = 3$.

Frequency 30; density 3

5. Cumulative frequency, quartiles and IQR

☐ I revised this

- Cumulative frequency is a running total. Plot each total at the **upper end** of its class and join with a smooth curve.
- Read the median at $n/2$, the lower quartile at $n/4$, the upper quartile at $3n/4$ (on the cumulative frequency axis).
- Interquartile range = UQ – LQ: the spread of the middle half, not affected by extremes.
- 'How many more than x ?' — read the cumulative frequency at x and subtract from the total.

How to do it

1. Make a running total column.
2. Plot (upper class boundary, running total).
3. Join with a smooth curve.
4. Read across from $n/4$, $n/2$, $3n/4$ and down to the x-axis.

Worked example

80 students: the graph gives LQ = 31 marks and UQ = 46 marks. Find the IQR. 52 students scored 40 or less: how many scored more than 40?

1. IQR = $46 - 31 = 15$.
2. $80 - 52 = 28$.

IQR 15 marks; 28 students

6. Scatter diagrams

☐ I revised this

- Correlation: positive (both increase), negative (one up, one down), or none.
- Line of best fit: one ruled line following the trend, with points roughly balanced either side; it need not pass through (0, 0).
- Use it to estimate within the data (interpolation). Estimates outside the data (extrapolation) are unreliable.
- An outlier is a point that doesn't fit the pattern.

Worked example

As temperature rises, sales of hot drinks fall. Describe the correlation.

1. One goes up while the other goes down.

Negative correlation

Common mistakes

Don't write	Write instead
✗ Median of 3, 4, 5, 7, 9, 12 is 5	✓ Even count: average the middle two, 6
✗ Giving the frequency as the modal class	✓ Give the class, e.g. 10–20
✗ Plotting cumulative frequency at midpoints	✓ Plot at the upper end of each class
✗ Histogram bar height = frequency with unequal widths	✓ Height = frequency density
✗ Dividing Σfx by the number of classes	✓ Divide by Σf , the total frequency

In the exam

- Show Σfx and Σf in a table — they earn method marks.
- When comparing data, compare an average AND a spread, and say what it means in context.
- Read graphs carefully — check the scale on each axis first.

My notes

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2D Trigonometry

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Pythagoras, SOH CAH TOA, elevation, depression and bearings, the sine and cosine rules, and $\frac{1}{2}ab \sin C$.

1. Pythagoras' theorem
2. Sin, cos and tan (SOH CAH TOA)
3. Elevation, depression and bearings
4. Sine rule and cosine rule
5. Area of any triangle

1. Pythagoras' theorem

☐ I revised this

- In a right-angled triangle: $a^2 + b^2 = c^2$, where c is the hypotenuse (opposite the right angle, the longest side).
- Finding the hypotenuse: square, **add**, square root. A shorter side: square, **subtract**, square root.
- Pythagorean triples to recognise: 3-4-5, 5-12-13, 8-15-17, 7-24-25 (and multiples like 6-8-10).

Formulas

$$a^2 + b^2 = c^2$$

c = hypotenuse

Worked example

A ladder 6.5 m long leans against a wall with its foot 2.5 m from the wall. How high up the wall does it reach?

1. Hypotenuse 6.5: $h^2 = 6.5^2 - 2.5^2$.
2. $= 42.25 - 6.25 = 36$.

6 m

2. Sin, cos and tan (SOH CAH TOA)

☐ I revised this

- Label the sides from the angle: **Opposite**, **Adjacent** (next to the angle), **Hypotenuse**.
- Choose the ratio that uses the two sides involved (one known, one wanted).
- Finding an angle: use the inverse ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$). Calculator in degrees.

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—

Formulas

$$\sin \theta = O/H$$

$$\cos \theta = A/H$$

$$\tan \theta = O/A$$

Worked example

In a right-angled triangle the angle is 52° and the adjacent side is 8 cm. Find the opposite side.

1. O and A $\rightarrow \tan$.
2. $O = 8 \times \tan 52^\circ = 10.239\dots$

10.2 cm (3 s.f.)

3. Elevation, depression and bearings

☐ I revised this

- **Angle of elevation:** measured up from the horizontal. **Angle of depression:** down from the horizontal.
- The angle of depression from A to B equals the angle of elevation from B to A (alternate angles).
- Bearings questions: draw north lines, mark the bearings, find a right-angled (or other) triangle.
- The shortest distance from a point to a line is the perpendicular distance.

Worked example

From a point 20 m from the foot of a tower, the angle of elevation of the top is 35° . Find the height.

1. $\tan 35^\circ = h \div 20$.
2. $h = 20 \times \tan 35^\circ = 14.0\dots$

14.0 m (3 s.f.)

4. Sine rule and cosine rule

☐ I revised this

- For triangles **without** a right angle. Label sides a, b, c opposite angles A, B, C.
- **Sine rule:** use when you know a side and its opposite angle (plus one more).
- **Cosine rule:** use with two sides and the angle between them (to find the third side) or all three sides (to find an angle).
- A negative cosine means an obtuse angle. $\sin x = \sin(180^\circ - x)$: the sine rule may give an obtuse answer (the ambiguous case).

Formulas

$a/\sin A = b/\sin B = c/\sin C$	sine rule
$a^2 = b^2 + c^2 - 2bc \cos A$	cosine rule (side)
$\cos A = (b^2 + c^2 - a^2) / 2bc$	cosine rule (angle)

Worked example

Triangle PQR: PQ = 6 cm, PR = 9 cm, angle P = 50° . Find QR.

1. $QR^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 50^\circ$.
2. $= 117 - 69.42 = 47.58$.

QR = 6.90 cm (3 s.f.)

5. Area of any triangle

☐ I revised this

- Area = $\frac{1}{2}ab \sin C$, where C is the angle **between** sides a and b.
- Use it backwards to find an angle or a side when the area is given.

Formulas

$$\text{Area} = \frac{1}{2}ab \sin C$$

Worked example

A triangle has sides 8 cm and 10 cm with 30° between them. Find its area.

1. $\frac{1}{2} \times 8 \times 10 \times \sin 30^\circ$.

2. $= 40 \times 0.5$.

20 cm²

Common mistakes

Don't write	Write instead
✗ Using SOH CAH TOA without a right angle	✓ Use the sine or cosine rule
✗ Forgetting to square root at the end of Pythagoras or the cosine rule	✓ $c^2 = 49 \rightarrow c = 7$
✗ Adding in Pythagoras when finding a shorter side	✓ Shorter side: subtract
✗ Calculator in radians	✓ Set it to degrees
✗ Giving only the acute angle when the triangle is obtuse	✓ The obtuse angle is $180^\circ - \text{acute}$

In the exam

- Draw and label the triangle, marking O, A, H or a, b, c, A, B, C.
- Keep full calculator values between steps; round only at the end (angles to 1 d.p.).
- Non-calculator paper: use the exact-value table.

My notes

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Mensuration

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Units, areas and perimeters, circles and sectors, surface area and volume of every solid, and working backwards.

Volumes of a cone, pyramid and sphere, and the curved surface areas of a cone and sphere, are on the formula list; the rest are not.

1. Units
2. Area and perimeter of 2D shapes
3. Circles, arcs, sectors and segments
4. Prisms, cuboids and cylinders
5. Pyramids, cones and spheres
6. Working backwards

1. Units

☐ I revised this

- Length: 1 km = 1000 m; 1 m = 100 cm; 1 cm = 10 mm.
- Area: square the conversion — $1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$; $1 \text{ cm}^2 = 100 \text{ mm}^2$.
- Volume: cube it — $1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$.
- Capacity: 1 litre = $1000 \text{ cm}^3 = 1000 \text{ ml}$; $1 \text{ cm}^3 = 1 \text{ ml}$.

Convert	Length	Area	Volume
m → cm	× 100	× 10 000	× 1 000 000
cm → mm	× 10	× 100	× 1000
km → m	× 1000	× 1 000 000	—

Worked example

Convert 3.5 m^2 to cm^2 .

1. $1 \text{ m}^2 = 10\,000 \text{ cm}^2$.
2. $3.5 \times 10\,000$.

$35\,000 \text{ cm}^2$

2. Area and perimeter of 2D shapes

☐ I revised this

- Perimeter = distance all the way round. Area = space inside.
- Use the **perpendicular** height for triangles, parallelograms and trapezia.
- Compound shapes: split into rectangles, triangles and semicircles, then add (or subtract a hole).

Formulas

rectangle $A = lw$

triangle $A = \frac{1}{2}bh$

parallelogram $A = bh$

trapezium $A = \frac{1}{2}(a + b)h$

Worked example

A semicircle of diameter 8 cm sits on top of an 8 cm × 5 cm rectangle. Find the total area.

1. Rectangle: $8 \times 5 = 40 \text{ cm}^2$.
2. Semicircle: $\frac{1}{2} \times \pi \times 4^2 = 8\pi = 25.13 \text{ cm}^2$.

65.1 cm² (3 s.f.)

3. Circles, arcs, sectors and segments

☐ I revised this

- Circumference = $\pi d = 2\pi r$. Area = πr^2 .
- A sector is $\theta/360$ of the circle: arc length = $\theta/360 \times 2\pi r$; sector area = $\theta/360 \times \pi r^2$.
- Perimeter of a sector = arc + 2 radii.
- Segment area = sector area – triangle area (triangle = $\frac{1}{2}r^2 \sin \theta$).

Formulas

$$C = 2\pi r$$

$$A = \pi r^2$$

$$\text{arc} = (\theta/360) \times 2\pi r$$

$$\text{sector} = (\theta/360) \times \pi r^2$$

Worked example

A sector has radius 10 cm and angle 90°. Find the area of the minor segment.

1. Sector: $\frac{1}{4} \times \pi \times 100 = 78.54 \text{ cm}^2$.
2. Triangle: $\frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$.
3. $78.54 - 50$.

28.5 cm² (3 s.f.)

Exam tip: 'In terms of π ' means leave π in: the sector area 12π , not 37.7.

4. Prisms, cuboids and cylinders

☐ I revised this

- Volume of a prism = area of cross-section × length. A cylinder is a prism with a circular cross-section.
- Surface area = total area of all faces (draw the net if unsure).
- Cylinder curved surface = $2\pi rh$ (the rectangle that wraps round); total = $2\pi rh + 2\pi r^2$.

Formulas

$$\text{prism } V = \text{cross-section area} \times \text{length}$$

$$\text{cylinder } V = \pi r^2 h$$

$$\text{cylinder curved SA} = 2\pi rh$$

Worked example

A cylinder has radius 3 cm and height 10 cm. Find its volume and total surface area in terms of π .

1. $V = \pi \times 9 \times 10 = 90\pi \text{ cm}^3$.
2. Curved: $2\pi \times 3 \times 10 = 60\pi$; two ends: $2 \times 9\pi = 18\pi$.

$V = 90\pi \text{ cm}^3$; $SA = 78\pi \text{ cm}^2$

5. Pyramids, cones and spheres

☐ I revised this

- Pyramid and cone volume = $\frac{1}{3} \times \text{base area} \times \text{height}$.
- Cone curved surface = $\pi r l$ (l = slant height; $l^2 = r^2 + h^2$).
- Sphere: $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$. Hemisphere: half of each, plus the flat circle πr^2 for total SA.
- Frustum = big cone – small cone.

Formulas

pyramid/cone $V = \frac{1}{3} \times \text{base} \times h$

cone curved $SA = \pi r l$

sphere $V = \frac{4}{3}\pi r^3$

sphere $SA = 4\pi r^2$

Worked example

A cylinder (radius 2 cm, height 5 cm) has a hemisphere of radius 2 cm on top. Find the volume.

- Cylinder: $\pi \times 4 \times 5 = 20\pi$.
- Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 8 = 16\pi/3$.
- Total = $76\pi/3 = 79.6$.

79.6 cm³ (3 s.f.)

Exam tip: The formulas for cone, pyramid and sphere are given on the formula list — but you must know which to use.

6. Working backwards

☐ I revised this

- Given the volume or area, substitute and solve for the missing length.
- Rearrange before using the calculator: $r^2 = V \div (\pi h)$, then square root.
- Equal volumes: liquid poured from one container into another keeps its volume.

Worked example

A cylinder has volume 500 cm³ and height 8 cm. Find its radius.

- $500 = \pi r^2 \times 8$.
- $r^2 = 500 \div (8\pi) = 19.89\dots$
- $r = \sqrt{19.89\dots}$

4.46 cm (3 s.f.)

Common mistakes

Don't write	Write instead
✗ Using the diameter as the radius	✓ Halve the diameter first
✗ Cone volume $\pi r^2 h$	✓ It's $\frac{1}{3}\pi r^2 h$
✗ $1 \text{ m}^2 = 100 \text{ cm}^2$	✓ $1 \text{ m}^2 = 10\,000 \text{ cm}^2$
✗ Sector perimeter = arc length only	✓ Add the two radii
✗ Using the slant height as h in a cone's volume	✓ Volume uses the vertical height

In the exam

- Write the formula, substitute, then calculate — each step can earn a mark.
- Keep π in until the end (or use the π key), then round.
- Check units: area in cm², volume in cm³.

My notes

Time

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Clocks, durations, decimal hours, timetables and time zones.

1. 12- and 24-hour clocks
2. Time intervals and decimal hours
3. Timetables and time zones

1. 12- and 24-hour clocks

☐ I revised this

- 24-hour times use four digits: 07 45, 19 45. Midnight is 00 00; noon is 12 00.
- pm → 24-hour: add 12 to the hours (except 12 pm). am times stay the same (12 am = 00 00).
- 24-hour → 12-hour: subtract 12 from hours above 12 and write pm.

12-hour	24-hour
12:30 am	00 30
7:45 am	07 45
12:15 pm	12 15
7:45 pm	19 45
11:59 pm	23 59

Worked example

Write 4:05 pm in the 24-hour clock.

1. pm and after 1 pm → add 12.
2. $4 + 12 = 16$.

16 05

2. Time intervals and decimal hours

☐ I revised this

- Count up on a time line: to the next whole hour, then whole hours, then the minutes left.
- Never subtract times like decimals: $21\ 15 - 18\ 50$ is not 2.65.
- Decimal hours → minutes: multiply the decimal part by 60. $2.35\ \text{h} = 2\ \text{h}\ 21\ \text{min}$.
- Minutes → decimal hours: divide by 60. $45\ \text{min} = 0.75\ \text{h}$; $48\ \text{min} = 0.8\ \text{h}$.

Time line method

1. From the start time, count to the next o'clock.
2. Count the whole hours.
3. Add the minutes at the end.
4. Add the pieces together.

Worked example

A bus leaves at 22 40 and arrives at 03 15 the next day. How long is the journey?

1. $22\ 40 \rightarrow 23\ 00$: 20 min.
2. $23\ 00 \rightarrow 03\ 00$: 4 h.
3. $03\ 00 \rightarrow 03\ 15$: 15 min.

4 h 35 min

3. Timetables and time zones

☐ I revised this

- Read a timetable along the row for the stop and down the column for one bus or train.
- Arrival = departure + journey time.
- A place that is **ahead** is later there: add the difference. Behind: subtract.
- Flight problems: work in one time zone first, then convert at the end.

Worked example

Karachi is 5 h ahead of London. A flight leaves London at 22 00 (London time) and takes 8 h. What is the local time in Karachi on landing?

1. Landing in London time: $22\ 00 + 8\ \text{h} = 06\ 00$.
2. Karachi is 5 h ahead: $06\ 00 + 5\ \text{h}$.

11 00 (next day)

Exam tip: Watch for journeys that cross midnight — the date changes too.

Common mistakes

Don't write	Write instead
✗ $2.35\ \text{h} = 2\ \text{h}\ 35\ \text{min}$	✓ $0.35 \times 60 = 21\ \text{min} \rightarrow 2\ \text{h}\ 21\ \text{min}$
✗ $21\ 15 - 18\ 50 = 2\ \text{h}\ 65\ \text{min}$	✓ Count up: $2\ \text{h}\ 25\ \text{min}$
✗ $12:30\ \text{am} = 12\ 30$	✓ $12:30\ \text{am}$ is just after midnight: $00\ 30$
✗ Using 100 minutes in an hour	✓ There are 60

In the exam

- Show your time line — it earns marks even if you slip in adding.
- Change minutes to hours ($\div 60$) before using them in a speed formula.
- Give answers in the form asked: 24-hour clock, hours and minutes, or just minutes.

My notes

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Kinematics

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Average speed, units, distance–time and speed–time graphs, acceleration, area as distance, and conversion graphs.

1. Speed, distance and time
2. Distance–time graphs
3. Speed–time graphs
4. Conversion graphs and rates

1. Speed, distance and time

☐ I revised this

- speed = distance $\div$ time; distance = speed $\times$ time; time = distance $\div$ speed.
- **Average speed** = total distance $\div$ total time, including stops. It is not the average of the speeds.
- Convert units first: 45 minutes = 0.75 h; km/h $\div$ 3.6 = m/s.

Formulas

speed = distance $\div$ time	
km/h $\rightarrow$ m/s	$\div$ 3.6
m/s $\rightarrow$ km/h	$\times$ 3.6

Worked example

A cyclist rides 30 km in 1 h, rests for 30 min, then rides 20 km in 1 h. Find the average speed.

1. Total distance 50 km.
2. Total time 2.5 h (the rest counts).
3. $50 \div 2.5$.

20 km/h

2. Distance–time graphs

☐ I revised this

- Gradient = speed. Steeper = faster.
- Horizontal line = stationary. Line sloping down = returning towards the start.
- A curve means the speed is changing; use a tangent for the speed at an instant.

Worked example

A straight line on a distance–time graph goes from (0 s, 0 m) to (40 s, 200 m). Find the speed.

1. Gradient = $200 \div 40$.

5 m/s

3. Speed–time graphs

☐ I revised this

- Gradient = acceleration (m/s²). Negative gradient = deceleration.
- Horizontal line = constant speed (not stopped!).
- **Area under the graph = distance travelled.** Split into triangles, rectangles and trapezia.

Formulas

acceleration = change in speed $\div$ time	gradient
distance = area under the graph	
trapezium area = $\frac{1}{2}(a + b)h$	

Worked example

A bus accelerates from 0 to 12 m/s in 8 s, keeps 12 m/s for 20 s, then stops in 6 s. How far does it go?

1. The shape is a trapezium with parallel sides 34 s and 20 s, height 12.
2. Area = $\frac{1}{2} \times (34 + 20) \times 12$.

324 m

4. Conversion graphs and rates

☐ I revised this

- A straight-line conversion graph through the origin means direct proportion: find the rate from one point.
- Use it to convert in either direction; for values off the graph, scale up.
- The gradient of any real-life graph is a rate (e.g. litres per minute, cost per kg).

Worked example

10 miles = 16 km. Convert 25 miles to km.

1. 1 mile = 1.6 km.
2. 25×1.6 .

40 km

Common mistakes

Don't write	Write instead
✗ 2 h 30 min = 2.3 h	✓ 30 min = 0.5 h, so 2.5 h
✗ Averaging the speeds of two parts	✓ Average speed = total distance $\div$ total time
✗ Reading distance from a speed–time graph's axis	✓ Distance is the AREA under it
✗ A flat line on a speed–time graph means stopped	✓ It means constant speed

In the exam

- Write the units: m/s, m/s², km/h.
- Split areas carefully and show each part.
- If the graph has different time units (minutes and hours), convert before calculating.

My notes

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Circles

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Circle vocabulary and every circle theorem, with the reasons examiners want.

1. Circle words
2. Right angles in circles
3. Angles at the centre and circumference
4. Alternate segment theorem
5. Writing reasons

1. Circle words

☐ I revised this

- **Radius:** centre to edge. **Diameter:** across through the centre ($2 \times$ radius).
- **Chord:** a line joining two points on the circle. **Tangent:** a line touching the circle at one point.
- **Arc:** part of the circumference. **Sector:** a 'pizza slice'. **Segment:** area between a chord and the arc.

Worked example

What is the line from the centre to the point where a tangent touches?

1. It is a radius.

A radius (it meets the tangent at 90°)

2. Right angles in circles

☐ I revised this

- The **angle in a semicircle** is 90° (angle at the circumference standing on a diameter).
- A **tangent meets the radius** at 90° .
- **Tangents from an external point** are equal in length (look for an isosceles triangle or a kite).
- The **perpendicular from the centre to a chord bisects** the chord.

Worked example

AB is a diameter and C is on the circle. Angle $CAB = 35^\circ$. Find angle CBA.

1. Angle $ACB = 90^\circ$ (angle in a semicircle).
2. $180 - 90 - 35$.

55°

3. Angles at the centre and circumference

☐ I revised this

- The **angle at the centre** is twice the angle at the circumference standing on the same arc.
- **Angles in the same segment** (standing on the same chord, same side) are equal.
- **Opposite angles of a cyclic quadrilateral** add to 180° .

Worked example

O is the centre. Angle $AOC = 140^\circ$. B is on the major arc and D on the minor arc. Find angle ADC.

1. Angle $ABC = 70^\circ$ (angle at the centre is twice the angle at the circumference).
2. ABCD is cyclic, so angle $ADC = 180 - 70$.

110°

4. Alternate segment theorem

☐ I revised this

- The angle between a tangent and a chord equals the angle in the **alternate segment** (on the other side of the chord).
- Spot it: a tangent, a chord from the point of contact, and a triangle drawn in the circle.

Worked example

A tangent at T makes 58° with chord TA. B is on the circle in the alternate segment. Find angle TBA.

1. Alternate segment theorem.

58°

5. Writing reasons

☐ I revised this

- Name the theorem every time: 'angle in a semicircle', 'angle at the centre is twice the angle at the circumference', 'opposite angles of a cyclic quadrilateral add to 180° ', 'tangent perpendicular to radius', 'alternate segment theorem'.
- Radii are equal $\rightarrow$ isosceles triangles: use them to find base angles.

Where to start

1. Mark radii and tangents on the diagram.
2. Look for right angles first.
3. Look for isosceles triangles.
4. Then centre/circumference, same segment, cyclic quadrilateral, alternate segment.

Worked example

Tangents from P touch the circle at A and B. Angle APB = 50° . Find angle PAB.

1. PA = PB (tangents from a point).
2. Triangle PAB is isosceles: $(180 - 50) \div 2$.

65°

Common mistakes

Don't write	Write instead
$\times$ Angle at circumference = $2 \times$ angle at centre	$\checkmark$ It's the other way: centre = $2 \times$ circumference
$\times$ Cyclic quadrilateral rules with the centre as a corner	$\checkmark$ All four corners must be on the circle
$\times$ Giving the angle without a reason	$\checkmark$ Each step needs the theorem named
$\times$ Assuming a line is a diameter	$\checkmark$ Only if it goes through the centre

In the exam

- Diagrams are not to scale — use theorems, never measure.
- Find the easy angles first (right angles, isosceles), then the theorems.
- Learn the exact wording of each theorem for the 'give a reason' marks.

My notes

Simultaneous

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Solving two equations together: elimination, substitution, word problems, and line-and-curve pairs.

1. Elimination
2. Substitution
3. Forming simultaneous equations
4. One linear and one non-linear equation

1. Elimination

☐ I revised this

- Make the coefficient of one letter the same in both equations (multiply one or both equations).
- **Same signs** → **subtract**; **different signs** → **add**.
- Solve for the remaining letter, then substitute back to find the other.
- Check both values in the equation you didn't substitute into.

How to do it

1. Label the equations (1) and (2).
2. Multiply to match one coefficient.
3. Add or subtract to eliminate it.
4. Solve; substitute back; check.

Worked example

Solve $3x + 2y = 19$ and $2x - 5y = -19$.

1. $(1) \times 5: 15x + 10y = 95$. $(2) \times 2: 4x - 10y = -38$.
2. Add: $19x = 57$, so $x = 3$.
3. $3(3) + 2y = 19 \rightarrow y = 5$.
4. Check (2): $6 - 25 = -19$ ✓

$x = 3, y = 5$

2. Substitution

☐ I revised this

- Best when one equation already says $y = \dots$ or $x = \dots$
- Replace that letter in the other equation, solve, then substitute back.

Worked example

Solve $y = 3x - 4$ and $2x + y = 11$.

1. $2x + (3x - 4) = 11$.
2. $5x = 15$, $x = 3$.
3. $y = 3(3) - 4 = 5$.

$x = 3, y = 5$

3. Forming simultaneous equations

☐ I revised this

- Choose a letter for each unknown and say what it means.
- Write one equation for each fact in the question.
- Answer in words with units at the end.

Worked example

2 adult tickets and 3 child tickets cost \$31. 1 adult and 1 child ticket cost \$12. Find the price of each.

1. $2a + 3c = 31$ and $a + c = 12$.
2. From the second, $a = 12 - c$.
3. $2(12 - c) + 3c = 31 \rightarrow c = 7, a = 5$.

Adult \$5, child \$7

4. One linear and one non-linear equation

☐ I revised this

- Rearrange the linear equation to $y = \dots$ (or $x = \dots$) and substitute into the non-linear one.
- This gives a quadratic: solve it to get two x values.
- Find the matching y for **each** x using the linear equation, and pair them.
- Graphically, the solutions are where the line meets the curve (0, 1 or 2 points).

Worked example

Solve $y = x + 1$ and $x^2 + y^2 = 25$.

1. $x^2 + (x + 1)^2 = 25 \rightarrow 2x^2 + 2x - 24 = 0$.
2. $x^2 + x - 12 = 0 \rightarrow (x + 4)(x - 3) = 0$.
3. $x = 3 \rightarrow y = 4; x = -4 \rightarrow y = -3$.

(3, 4) and (-4, -3)

Common mistakes

Don't write	Write instead
 Subtracting when the signs are different	 Different signs: add the equations
 $-2y - (-2y) = -4y$	 $-2y - (-2y) = 0$
 Giving only the x values	 Find each y and pair it with its x
 Squaring $(x + 1)^2$ as $x^2 + 1$	 $(x + 1)^2 = x^2 + 2x + 1$

In the exam

- Write what you do to each equation: $(1) \times 2, (2) \times 3$.
- Check your pair in the original equations — a quick way to catch sign errors.
- In word problems, state the answer in words with units.

My notes

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Co-Ordinate Geometry

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Coordinates, gradient, length and midpoint, $y = mx + c$, and parallel and perpendicular lines.

1. Coordinates and straight-line graphs
2. Gradient
3. Length and midpoint
4. The equation $y = mx + c$
5. Parallel and perpendicular lines

1. Coordinates and straight-line graphs

☐ I revised this

- (x, y) : across first, then up. Points can be in any of the four quadrants.
- Draw a line from its equation with a table of three x values (three points catch mistakes).
- $x = a$ is a **vertical** line; $y = b$ is a **horizontal** line.
- A line crosses the y -axis where $x = 0$ and the x -axis where $y = 0$.

Worked example

Find where $3x + 2y = 12$ crosses the axes.

1. $x = 0$: $2y = 12 \rightarrow y = 6$.

2. $y = 0$: $3x = 12 \rightarrow x = 4$.

(0, 6) and (4, 0)

2. Gradient

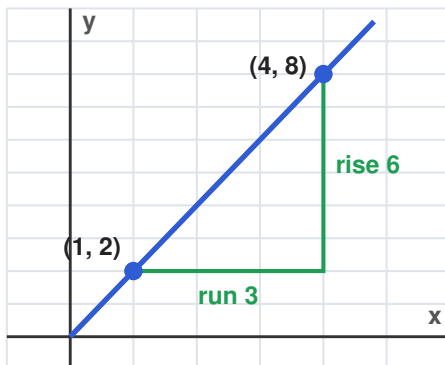
☐ I revised this

- Gradient = change in $y \div$ change in x (rise over run).
- Uphill left to right $\rightarrow$ positive; downhill $\rightarrow$ negative; horizontal $\rightarrow 0$; vertical $\rightarrow$ undefined.
- From a graph: pick two points exactly on grid corners, far apart.

Formulas

$$m = (y_2 - y_1) / (x_2 - x_1)$$

gradient between two points



Rise 6, run 3: gradient $6 \div 3 = 2$.

Worked example

Find the gradient of the line through $(-1, 7)$ and $(3, -5)$.

1. Change in y : $-5 - 7 = -12$.
2. Change in x : $3 - (-1) = 4$.
3. $-12 \div 4$.

-3

3. Length and midpoint

☐ I revised this

- Midpoint: average the x 's and average the y 's.
- Length: draw the right-angled triangle and use Pythagoras on the run and the rise.

Formulas

$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$	midpoint
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	length

Worked example

A is $(-2, 1)$ and B is $(4, 9)$. Find the midpoint and the length of AB.

1. Midpoint: $((-2 + 4)/2, (1 + 9)/2) = (1, 5)$.
2. Run 6, rise 8: $\sqrt{(36 + 64)} = \sqrt{100}$.

Midpoint (1, 5); length 10

4. The equation $y = mx + c$

☐ I revised this

- m is the gradient; c is where the line cuts the y -axis.
- Rearrange to $y = \dots$ before reading m : $2y = 6x + 4 \rightarrow y = 3x + 2$, so $m = 3$.
- Through a point with gradient m : substitute the point into $y = mx + c$ to find c (or use $y - y_1 = m(x - x_1)$).
- Through two points: find m first, then c .

How to do it

1. Find m (from two points if needed).
2. Substitute one point into $y = mx + c$.
3. Solve for c .
4. Write $y = mx + c$.

Worked example

Find the equation of the line through $(2, 7)$ and $(5, 13)$.

1. $m = (13 - 7) \div (5 - 2) = 2$.
2. $7 = 2(2) + c \rightarrow c = 3$.

$y = 2x + 3$

5. Parallel and perpendicular lines

☐ I revised this

- Parallel lines have the **same gradient**.
- Perpendicular gradients multiply to -1 : flip the fraction and change the sign ($2/3 \rightarrow -3/2$).
- Perpendicular bisector of AB: passes through the midpoint of AB, with gradient perpendicular to AB.

Formulas

$$m_1 = m_2$$

parallel

$$m_1 \times m_2 = -1$$

perpendicular

Worked example

A(1, 2) and B(5, 4). Find the equation of the perpendicular bisector of AB.

1. Midpoint (3, 3).
2. Gradient of AB = $2/4 = 1/2$, so perpendicular gradient -2 .
3. $3 = -2(3) + c \rightarrow c = 9$.

$$y = -2x + 9$$

Common mistakes

Don't write	Write instead
$\times$ Run over rise: $(x_2 - x_1)/(y_2 - y_1)$	$\checkmark$ Always y on top: change in y $\div$ change in x
$\times$ Reading m from $2y = 6x + 4$ as 6	$\checkmark$ Make y the subject first: $y = 3x + 2$, so $m = 3$
$\times$ Perpendicular to $m = 2$ written as -2	$\checkmark$ Flip it too: $-1/2$
$\times$ $3 - (-2) = 1$	$\checkmark$ $3 - (-2) = 5$. Put negatives in brackets

In the exam

- None of the gradient, midpoint or length formulas are given — learn them.
- Sketch the points roughly first: it shows whether the gradient should be positive or negative.
- Leave a length as a surd if the question asks for an exact answer.

My notes

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Number Sequences

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Term-to-term rules, nth terms of linear, special, quadratic, cubic and exponential sequences, and shape patterns.

1. Term-to-term rules
2. Linear sequences: the nth term
3. Special sequences
4. Quadratic sequences
5. Cubic and exponential sequences
6. Patterns of shapes

1. Term-to-term rules

☐ I revised this

- The term-to-term rule says how to get from one term to the next: 3, 7, 11, 15 → add 4.
- Arithmetic (linear) sequences add the same each time; geometric ones multiply by the same each time.
- Look at the differences: constant first difference → linear; constant second difference → quadratic.

Worked example

Find the next two terms of 2, 6, 18, 54, ...

1. Each term $\times 3$.

162, 486

2. Linear sequences: the nth term

☐ I revised this

- nth term = $dn + (\text{first term} - d)$, where d is the common difference.
- To test whether a number is a term, set the nth term equal to it: a whole-number n means yes.
- To find which term a value is, solve nth term = value.

Formulas

nth term = $dn + (a - d)$

a = first term, d = difference

Worked example

Find the nth term of 7, 11, 15, 19, ... Is 101 a term?

1. $d = 4$: $4n$. $4n$ gives 4, 8, 12...; add 3.
2. nth term $4n + 3$.
3. $4n + 3 = 101 \rightarrow n = 24.5$, not a whole number.

$4n + 3$; 101 is not a term

3. Special sequences

☐ I revised this

- Recognise these and their shifts (+ or -) and multiples:

Sequence	Terms	nth term
Square	1, 4, 9, 16, 25	n^2
Cube	1, 8, 27, 64, 125	n^3
Triangle	1, 3, 6, 10, 15	$n(n + 1)/2$
Powers of 2	2, 4, 8, 16, 32	2^n
Square + 1	2, 5, 10, 17, 26	$n^2 + 1$
Twice square	2, 8, 18, 32, 50	$2n^2$

Worked example

Find the nth term of 0, 3, 8, 15, 24, ...

1. Compare with squares 1, 4, 9, 16, 25: each is 1 less.

$$n^2 - 1$$

4. Quadratic sequences

☐ I revised this

- If the **second** difference is constant, the nth term is $an^2 + bn + c$.
- $a = \text{second difference} \div 2$.

How to do it

1. Find the first and second differences.
2. $a = \text{second difference} \div 2$.
3. Subtract an^2 from each term.
4. Find the nth term of what is left (linear) and add it on.

Worked example

Find the nth term of 4, 10, 18, 28, 40, ...

1. Differences 6, 8, 10, 12; second difference 2, so $a = 1$.
2. Subtract n^2 (1, 4, 9, 16, 25): 3, 6, 9, 12, 15 = $3n$.

$$n^2 + 3n$$

5. Cubic and exponential sequences

☐ I revised this

- Constant **third** difference → cubic; often a simple form like $n^3 + 1$.
- A constant **ratio** → exponential: nth term = $a \times r^{n-1}$ ($a = \text{first term}$, $r = \text{ratio}$).
- Combinations: compare with the basic sequence and spot the change.

Formulas

$$\text{nth term} = a \times r^{n-1}$$

geometric (exponential)

Worked example

Find the nth term of 5, 10, 20, 40, ...

1. Ratio $r = 2$, first term 5.

$$5 \times 2^{n-1}$$

6. Patterns of shapes

☐ I revised this

- Count the items in the first few patterns and write them as a sequence.
- Find the n th term, then use it to predict any pattern or to find which pattern uses a given number.

Worked example

Pattern n uses $3n + 1$ matchsticks. Which pattern uses 100 matchsticks?

1. $3n + 1 = 100$.

2. $3n = 99$.

Pattern 33

Common mistakes

Don't write	Write instead
✗ n th term of 5, 8, 11 is $n + 3$	✓ The difference is 3, so $3n + 2$
✗ Using the second difference as a	✓ a is HALF the second difference
✗ Saying 101 is the 24.5th term	✓ n must be a whole number, so 101 is not a term
✗ 5×2^n for 5, 10, 20, 40	✓ Check $n = 1$: $5 \times 2^{n-1}$ gives 5

In the exam

- Always check your n th term with $n = 1$ and $n = 2$.
- Show the differences — they earn method marks.
- Give the n th term in simplest form ($3n + 2$, not $3n + 5 - 3$).

My notes

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Graphical Solutions

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Solving equations with graphs, finding the line to draw, simultaneous equations and gradients from tangents.

1. Solving equations from a graph
2. Finding the line to draw
3. Simultaneous equations on a graph
4. Gradient of a curve

1. Solving equations from a graph

☐ I revised this

- Solutions of $f(x) = 0$ are the x values where the curve crosses the **x-axis**.
- Solutions of $f(x) = k$: draw the horizontal line $y = k$ and read the x values where it meets the curve.
- The number of crossing points is the number of solutions.

Worked example

The graph of $y = x^2 - x - 6$ is drawn. Solve $x^2 - x - 6 = 4$.

1. Draw $y = 4$.
2. Read the x values where the line meets the curve.

about $x = -2.7$ and $x = 3.7$

2. Finding the line to draw

☐ I revised this

- You have the graph of one curve and must solve a different equation.
- Rearrange the new equation so one side is **exactly** the curve you have; the other side is the line to draw.

How to do it

1. Write the curve's expression (e.g. $x^2 - 3x$).
2. Rearrange the new equation to: that expression = something.
3. Draw $y = \text{something}$.
4. Read the x values of the crossing points.

Worked example

The graph of $y = x^2 - 3x$ is drawn. What line should you draw to solve $x^2 - 5x + 2 = 0$?

1. Add $2x - 2$ to both sides: $x^2 - 3x = 2x - 2$.

Draw $y = 2x - 2$

Exam tip: Check by subtracting: $x^2 - 3x - (2x - 2)$ should give back $x^2 - 5x + 2$.

3. Simultaneous equations on a graph

☐ I revised this

- The solution is the point where the two graphs cross: read both x and y .
- A line and a curve can cross 0, 1 or 2 times (a cubic up to 3).

Worked example

Where do $y = 2x + 1$ and $y = 7 - x$ meet?

1. $2x + 1 = 7 - x \rightarrow x = 2$.

2. $y = 2(2) + 1 = 5$.

(2, 5)

4. Gradient of a curve

☐ I revised this

- Draw a **tangent** at the point: a straight line that touches the curve there without cutting it.
- Pick two points far apart on the tangent; gradient = rise $\div$ run.
- On a distance–time graph the gradient is speed; on a speed–time graph it is acceleration.

How to do it

1. Draw the tangent with a ruler.
2. Choose two points on it, far apart.
3. Gradient = change in $y \div$ change in x .

Worked example

A tangent passes through (1, 3) and (4, 12). Find its gradient.

1. Rise = $12 - 3 = 9$.

2. Run = $4 - 1 = 3$.

Gradient 3

Common mistakes

Don't write	Write instead
 Reading y values when x values are asked	 Solutions of an equation in x are x values
 Drawing $y = x - 3$ for $x^2 + x - 3 = 0$ with $y = x^2 + 2x$	 Add the same to both sides: $y = x + 3$
 Using two points on the curve for a gradient	 Use two points on the tangent
 A tangent that cuts through the curve	 It must touch at one point only

In the exam

- Draw lines long enough to meet the curve everywhere it can.
- State the equation of any line you draw.
- Gradient estimates are given a range of accepted answers — draw carefully and read accurately.

My notes

Linear Inequalities And Shading

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Inequality symbols, number lines, solving inequalities, and regions on a graph.

1. Symbols and number lines
2. Solving inequalities
3. Regions on a graph

1. Symbols and number lines

☐ I revised this

- $<$ less than, $>$ greater than, $\leq$ less than or equal to, $\geq$ greater than or equal to.
- On a number line: open circle for $<$ or $>$; filled circle for $\leq$ or $\geq$.
- Double inequality $-2 < x \leq 3$ is drawn between the two values: open at -2 , filled at 3 .

Inequality	Circle	Arrow
$x > 2$	open at 2	right
$x \geq 2$	filled at 2	right
$x < -1$	open at -1	left
$-2 < x \leq 3$	open at -2 , filled at 3	between

Worked example

List the integers n with $-2 < n \leq 3$.

1. -2 is not included; 3 is.

$-1, 0, 1, 2, 3$

2. Solving inequalities

☐ I revised this

- Solve like an equation: add, subtract, multiply or divide both sides.
- **Multiplying or dividing by a negative number reverses the sign.**
- Double inequalities: do the same to all three parts.

Worked example

Solve $-3 \leq 2x + 1 < 7$ and list the integer solutions.

1. Subtract 1: $-4 \leq 2x < 6$.
2. Divide by 2: $-2 \leq x < 3$.

$x = -2, -1, 0, 1, 2$

Exam tip: To avoid flipping, move the x term to the side where it is positive: $4 - 2x \leq 10 \rightarrow -6 \leq 2x \rightarrow x \geq -3$.

3. Regions on a graph

☐ I revised this

- Draw each boundary line: **solid** for $\leq$ or $\geq$, **broken** for $<$ or $>$.
- Pick a test point not on the line (usually $(0, 0)$) to decide which side you want.
- Read the question: shade the region you want, or the region you **don't** want (unwanted region).
- $y \geq 1$ is above the line $y = 1$; $x \leq 3$ is to the left of $x = 3$.

How to do it

1. Draw the line (make a table or use $y = mx + c$).
2. Decide solid or broken.
3. Test a point: does it make the inequality true?
4. Shade the correct side.

Worked example

The region R satisfies $x \geq 1$, $y \geq 2$ and $x + y \leq 8$. Find the greatest value of $3x + 2y$ for integer points in R.

1. Corners of R: (1, 2), (6, 2), (1, 7).
2. $3x + 2y$: 7, 22 and 17.

22, at (6, 2)

Exam tip: Maximum and minimum values of an expression in a region are always at or near a corner.

Common mistakes

Don't write	Write instead
$\times -2x \leq 6 \rightarrow x \leq -3$	$\checkmark$ Dividing by -2 flips the sign: $x \geq -3$
$\times$ Including -2 in $-2 < n$	$\checkmark$ $<$ means -2 is not included
$\times$ Drawing $y < x + 3$ with a solid line	$\checkmark$ Strict inequality $\rightarrow$ broken line
$\times$ Shading the wanted region when the question says shade the unwanted one	$\checkmark$ Read which region to shade

In the exam

- Label every boundary line with its equation.
- Use (0, 0) as a test point when it is not on the line — the arithmetic is easy.
- When listing integers, check the end values against the symbols.

My notes

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Symmetry

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Lines of symmetry, rotational symmetry, symmetric quadrilaterals and planes of symmetry in solids.

1. Line symmetry
2. Rotational symmetry
3. Symmetry of triangles and quadrilaterals
4. Symmetry in solids

1. Line symmetry

☐ I revised this

- A line of symmetry splits a shape into two mirror images.
- Test by folding (or with a mirror or tracing paper).
- A regular polygon with n sides has n lines of symmetry.

Worked example

How many lines of symmetry does a regular hexagon have?

1. Regular n -gon: n lines.

6

2. Rotational symmetry

☐ I revised this

- The **order** is how many times a shape looks the same in one full turn.
- Order 1 means no rotational symmetry (it only matches itself after 360°).
- A regular n -gon has rotational symmetry of order n .

Worked example

What is the order of rotational symmetry of a parallelogram?

1. It looks the same after a half turn (180°) and a full turn.

Order 2

3. Symmetry of triangles and quadrilaterals

☐ I revised this

- Know these well — they are used to find missing angles and lengths.

Shape	Lines	Order
Equilateral triangle	3	3
Isosceles triangle	1	1
Square	4	4
Rectangle	2	2
Rhombus	2 (the diagonals)	2
Parallelogram	0	2
Kite	1	1
Isosceles trapezium	1	1

Worked example

Which quadrilateral has rotational symmetry of order 2 but no lines of symmetry?

1. Rectangle and rhombus have lines; parallelogram has none.

Parallelogram

4. Symmetry in solids

☐ I revised this

- A **plane of symmetry** cuts a solid into two mirror halves.
- Cuboid (all edges different): 3 planes. Cube: 9 planes.
- Prisms, cylinders, pyramids and cones: picture slicing through the middle.

Worked example

How many planes of symmetry does a cuboid with three different edge lengths have?

1. One plane through the middle of each pair of opposite faces.

3

Common mistakes

Don't write	Write instead
✗ A parallelogram has 2 lines of symmetry	✓ It has none; its order of rotation is 2
✗ Order 0 rotational symmetry	✓ The smallest order is 1
✗ A rhombus's lines go through the midpoints of its sides	✓ Its lines are the diagonals

In the exam

- Use tracing paper in the exam to check lines and order.
- Complete-the-pattern questions: reflect each square across the given line(s).
- Answer with a number for 'order' — 'order 2', not 'twice'.

My notes

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Polygons

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Angle facts, parallel lines, angles in polygons, and how to write geometric reasons.

1. Basic angle facts
2. Angles and parallel lines
3. Angles in polygons
4. Giving reasons

1. Basic angle facts

☐ I revised this

- Angles on a straight line add to 180° . Angles at a point add to 360° .
- Vertically opposite angles are equal.
- Angles in a triangle add to 180° ; in a quadrilateral, 360° .
- Isosceles triangle: two equal sides and the two base angles equal. Equilateral: all angles 60° .
- The exterior angle of a triangle equals the sum of the two opposite interior angles.

Worked example

Three angles on a straight line are $2x$, $3x$ and $4x$. Find x .

1. $2x + 3x + 4x = 180$.

2. $9x = 180$.

$x = 20$

2. Angles and parallel lines

☐ I revised this

- **Alternate** angles (Z-shape) are equal.
- **Corresponding** angles (F-shape) are equal.
- **Co-interior** angles (C-shape) add to 180° .
- Write the reason in words: 'alternate angles are equal' — not 'Z angles'.

Pair	Shape	Rule
Alternate	Z	equal
Corresponding	F	equal
Co-interior	C / U	add to 180°
Vertically opposite	X	equal

Worked example

Two parallel lines are crossed by a transversal. One co-interior angle is 115° . Find the other.

1. Co-interior angles add to 180° .

2. $180 - 115$.

65°

3. Angles in polygons

☐ I revised this

- Sum of interior angles of an n -sided polygon = $(n - 2) \times 180^\circ$.
- Exterior angles of any polygon add to 360° .
- Regular polygon: exterior angle = $360^\circ \div n$; interior = $180^\circ - \text{exterior}$.
- Number of sides of a regular polygon = $360^\circ \div \text{exterior angle}$.

Polygon	Sides	Angle sum	Regular interior
Triangle	3	180°	60°
Quadrilateral	4	360°	90°
Pentagon	5	540°	108°
Hexagon	6	720°	120°
Octagon	8	1080°	135°
Decagon	10	1440°	144°

Formulas

$(n - 2) \times 180^\circ$	interior angle sum
$360^\circ \div n$	exterior angle of a regular polygon
interior + exterior = 180°	

Worked example

The interior angle of a regular polygon is 5 times its exterior angle. How many sides does it have?

1. Interior + exterior = 180 , so $6 \times \text{exterior} = 180$.
2. Exterior = 30° .
3. $n = 360 \div 30$.

12 sides

4. Giving reasons

☐ I revised this

- Use three-letter notation: angle ABC is the angle at B.
- Each step: the angle, then its reason. For example: 'angle ABD = 40° (alternate angles are equal)'.
- Common reasons: angles on a straight line, angles in a triangle, isosceles triangle, alternate / corresponding / co-interior angles, angles in a polygon.

Worked example

In triangle ABC, AB = AC and angle BAC = 40° . Find angle ABC, giving a reason.

1. Base angles of an isosceles triangle are equal.
2. $(180 - 40) \div 2 = 70$.

70° (base angles of an isosceles triangle are equal; angles in a triangle add to 180°)

Common mistakes

Don't write	Write instead
$\times$ Regular hexagon interior angle = $360 \div 6 = 60^\circ$	$\checkmark$ That's the exterior angle; interior = 120°
$\times$ Using $n \times 180$ for the angle sum	$\checkmark$ $(n - 2) \times 180$
$\times$ Writing 'Z angles' as a reason	$\checkmark$ Write 'alternate angles are equal'
$\times$ Saying co-interior angles are equal	$\checkmark$ They add to 180°

In the exam

- Mark angles on the diagram as you find them.
- 'Give reasons' questions lose marks without the words — write every reason.
- Diagrams are not drawn to scale: never measure; calculate.

My notes

Daily Maths

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Everyday maths: percentages, reverse percentages, interest, ratio, scales, rates, units and pie charts.

1. Percentages
2. Reverse percentages
3. Simple and compound interest
4. Ratio and proportion
5. Rates, currency and units
6. Pie charts

1. Percentages

☐ I revised this

- Percentage of an amount: change the % to a decimal and multiply. 35% of 240 = $0.35 \times 240 = 84$.
- One quantity as a percentage of another: $(\text{part} \div \text{whole}) \times 100$.
- Use a **multiplier** for increases and decreases: +15% $\rightarrow \times 1.15$; -15% $\rightarrow \times 0.85$.
- Percentage change = $\text{change} \div \text{original} \times 100$ (profit and loss % work the same way).

Formulas

percentage change = $(\text{change} \div \text{original}) \times 100$

new = original $\times$ multiplier

Worked example

A price rises from \$40 to \$46. Find the percentage increase.

1. Change = 6.
2. $6 \div 40 = 0.15$.

15%

2. Reverse percentages

☐ I revised this

- You know the amount **after** a percentage change and want the original.
- Original = new amount $\div$ multiplier. After a 20% discount the multiplier is 0.8.
- Never take the percentage of the new amount.

How to do it

1. Write the multiplier for the change (e.g. +15% $\rightarrow 1.15$).
2. Divide the new amount by it.

Worked example

A phone costs \$345 after a 15% price rise. What did it cost before?

1. Multiplier = 1.15.
2. $345 \div 1.15$.

\$300

3. Simple and compound interest

☐ I revised this

- **Simple** interest is the same every year: $I = PRT \div 100$ (P = principal, R = rate %, T = years).
- **Compound** interest is added to the amount each year, so it grows: $A = P(1 + r/100)^n$.
- Total amount with simple interest = $P + I$. Compound interest earned = $A - P$.

Formulas

$I = PRT/100$	simple interest
$A = P(1 + r/100)^n$	compound: amount after n years

Worked example

\$2000 is invested at 5% per year compound interest for 3 years. Find the interest earned.

1. $A = 2000 \times 1.05^3 = 2315.25$.
2. Interest = $2315.25 - 2000$.

\$315.25

4. Ratio and proportion

☐ I revised this

- Simplify a ratio by dividing all parts by the HCF; use the same units first.
- Sharing in a ratio: add the parts, find one part, multiply.
- Map scale 1 : 50 000 means 1 cm on the map = 50 000 cm = 500 m in real life.
- Best value: compare the price of the same amount (per 100 g, per litre) or the amount per \$1.
- Recipes: find the amount for one person (or one unit), then scale up.

Worked example

Share \$360 in the ratio 4 : 5.

1. $4 + 5 = 9$ parts.
2. One part = $360 \div 9 = 40$.
3. $4 \times 40 = 160$ and $5 \times 40 = 200$.

\$160 and \$200

5. Rates, currency and units

☐ I revised this

- Exchange rates: multiply to change into the currency the rate is quoted in, divide to change back.
- Rates: speed (km/h), hourly pay (\$/h), flow (litres/min), fuel use (km/litre), density (g/cm³), population density (people/km²).
- Length: 1 km = 1000 m, 1 m = 100 cm, 1 cm = 10 mm. Mass: 1 kg = 1000 g; 1 tonne = 1000 kg.
- Area and volume scale with the square and cube: 1 m² = 10 000 cm²; 1 m³ = 1 000 000 cm³; 1 litre = 1000 cm³.

Formulas

density = mass $\div$ volume
population density = population $\div$ area

Worked example

\$1 = 280 rupees. Convert 7000 rupees to dollars.

1. Rupees $\rightarrow$ dollars: divide by 280.
2. $7000 \div 280$.

\$25

6. Pie charts

☐ I revised this

- The whole pie is 360° . Angle for a group = frequency $\div$ total $\times 360^\circ$.
- From a pie chart: frequency = angle $\div 360 \times$ total.
- If you know one sector's frequency and angle, find 'degrees per item' and scale.

Worked example

In a survey of 72 students, 18 walk to school. What angle represents walking?

1. $18 \div 72 = 1/4$.

2. $1/4 \times 360^\circ$.

90°

Common mistakes

Don't write	Write instead
$\times$ Reverse %: $64 + 20\%$ of 64	$\checkmark$ Divide by the multiplier: $64 \div 0.8 = 80$
$\times$ Percentage change $\div$ the new value	$\checkmark$ Always divide by the original value
$\times$ Compound interest worked as simple interest	$\checkmark$ Multiply by 1.05 each year: $P \times 1.05^n$
$\times$ $1 \text{ m}^2 = 100 \text{ cm}^2$	$\checkmark$ $1 \text{ m}^2 = 100 \times 100 = 10\,000 \text{ cm}^2$
$\times$ Sharing in 4 : 5 by dividing by 4 and 5	$\checkmark$ Divide by the total number of parts, 9

In the exam

- Money answers: give 2 decimal places (\$315.25, not \$315.3).
- Say clearly what you are comparing in best-value questions (price per 100 g...).
- Read whether the question wants the interest or the total amount.

My notes

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Proportionality

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Direct and inverse proportion with $\propto$ and k , including squares, cubes and roots.

1. Direct proportion
2. Inverse proportion
3. The effect of a change

1. Direct proportion

☐ I revised this

- $y \propto x$ means $y = kx$: when x doubles, y doubles. The graph is a straight line through the origin.
- Other direct forms: $y \propto x^2$ ($y = kx^2$), $y \propto x^3$, $y \propto \sqrt{x}$, $y \propto \sqrt[3]{x}$.
- Find k from a given pair of values, then use the formula.

Method for every proportion question

1. Write the formula with k .
2. Substitute the pair you are given to find k .
3. Write the formula again with the number for k .
4. Substitute the new value.

Formulas

$$y \propto x \Rightarrow y = kx$$

$$y \propto x^2 \Rightarrow y = kx^2$$

$$y \propto \sqrt{x} \Rightarrow y = k\sqrt{x}$$

Worked example

y is directly proportional to the square of x . $y = 18$ when $x = 3$. Find y when $x = 5$.

1. $y = kx^2$.
2. $18 = k \times 9$, so $k = 2$.
3. $y = 2x^2$.
4. $y = 2 \times 25$.

$y = 50$

2. Inverse proportion

☐ I revised this

- $y \propto 1/x$ means $y = k/x$: when x doubles, y halves. xy is always the same.
- Other inverse forms: $y \propto 1/x^2$, $y \propto 1/\sqrt{x}$.
- The graph of $y = k/x$ is a curve that gets close to both axes.

Formulas

$$y \propto 1/x \Rightarrow y = k/x$$

$$y \propto 1/x^2 \Rightarrow y = k/x^2$$

Worked example

The force F is inversely proportional to the square of the distance d . $F = 20$ when $d = 3$. Find F when $d = 6$.

1. $F = k/d^2$.
2. $20 = k/9$, so $k = 180$.
3. $F = 180/36$.

$F = 5$

3. The effect of a change

☐ I revised this

- If $y \propto x^n$ and x is multiplied by m , then y is multiplied by m^n .
- Inverse: if $y \propto 1/x^n$ and x is multiplied by m , y is divided by m^n .
- Percentage changes work the same way: x up 10% means $x \times 1.1$.

Relationship	x doubled $\rightarrow y$	x tripled $\rightarrow y$
$y \propto x$	$\times 2$	$\times 3$
$y \propto x^2$	$\times 4$	$\times 9$
$y \propto x^3$	$\times 8$	$\times 27$
$y \propto 1/x$	$\div 2$	$\div 3$
$y \propto 1/x^2$	$\div 4$	$\div 9$

Worked example

$y \propto x^3$. x is increased by 10%. By what percentage does y increase?

1. x is multiplied by 1.1.
2. y is multiplied by $1.1^3 = 1.331$.

33.1%

Common mistakes

Don't write	Write instead
 Using $y = kx$ for 'proportional to the square'	 Square it: $y = kx^2$
 Inverse proportion written $y = x/k$	 It is $y = k/x$
 Not squaring when substituting into $y = kx^2$	 $18 = k \times 3^2$, not $18 = k \times 3$
 Forgetting to write the formula with k	 Write $y = 2x^2$ — it is often worth a mark

In the exam

- Read the wording carefully: 'square', 'square root', 'cube', 'inversely'.
- Always write the formula with your value of k in it before the final step.
- Check: a larger x should make y bigger (direct) or smaller (inverse).

My notes

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Limits Of Accuracy

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Upper and lower bounds of rounded values and of calculations that use them.

1. Bounds of a rounded value
2. Bounds of calculations
3. Giving an answer to a suitable accuracy

1. Bounds of a rounded value

☐ I revised this

- A measurement rounded to some accuracy could have been anything within **half a unit** either side.
- Lower bound = value – half the unit; upper bound = value + half the unit.
- The upper bound itself would round up, so the error interval is **lower** $\leq x <$ **upper**.

Given as	Unit	Lower bound	Upper bound
7.4 cm (1 d.p.)	0.1	7.35	7.45
3400 (nearest 100)	100	3350	3450
12 kg (nearest kg)	1	11.5	12.5
0.36 (2 s.f.)	0.01	0.355	0.365

Worked example

A length is 7.4 cm to the nearest mm. Write its error interval.

1. Nearest mm = nearest 0.1 cm.
2. Half of 0.1 = 0.05.

$7.35 \leq l < 7.45$

2. Bounds of calculations

☐ I revised this

- To make an answer as **large** as possible, choose the bounds that push it up; for as **small** as possible, the opposite.
- Adding and multiplying: upper with upper gives the upper bound.
- Subtracting and dividing: one of each. The biggest difference is upper – lower; the biggest quotient is upper $\div$ lower.

Calculation	Upper bound	Lower bound
$a + b$	$Ua + Ub$	$La + Lb$
$a \times b$	$Ua \times Ub$	$La \times Lb$
$a - b$	$Ua - Lb$	$La - Ub$
$a \div b$	$Ua \div Lb$	$La \div Ub$

Worked example

A car travels 120 m (nearest 10 m) in 15.2 s (1 d.p.). Find the lower bound of its speed.

1. Lower distance = 115 m; upper time = 15.25 s.
2. Lower speed = $115 \div 15.25 = 7.540\dots$

7.54 m/s (3 s.f.)

Exam tip: Write the bounds you are using before calculating — they earn method marks.

3. Giving an answer to a suitable accuracy

☐ I revised this

- Work out both the upper and lower bound of the result.
- Round both to fewer and fewer figures until they agree: that is the accuracy you can trust.
- Example: bounds 7.54 and 7.81 agree only to 1 s.f. → answer 8.

Worked example

The bounds of a length are 4.4637 m and 4.4762 m. Give the length to a suitable accuracy.

1. To 3 s.f.: 4.46 and 4.48 — different.

2. To 2 s.f.: 4.5 and 4.5 — the same.

4.5 m

Common mistakes

Don't write	Write instead
✗ Upper bound of 7.4 (1 d.p.) is 7.49	✓ It is 7.45 — the boundary
✗ Error interval $11.5 \leq x \leq 12.5$	✓ Upper end is strict: $11.5 \leq x < 12.5$
✗ Upper bound of $a \div b = Ua \div Ub$	✓ Divide by the smallest: $Ua \div Lb$
✗ Upper bound of $a - b = Ua - Ub$	✓ Subtract the smallest: $Ua - Lb$

In the exam

- Find the unit first ('nearest 5' → half unit 2.5; '2 s.f.' on 360 → nearest 10).
- Keep full calculator values for bounds; round only the final answer.
- For a mixed calculation (like speed = d/t or density = m/V) decide each bound separately.

My notes

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Hcf Lcm

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Prime factors, HCF and LCM, the problems they solve, and using primes for roots.

1. Prime factorisation
2. HCF and LCM
3. Word problems
4. Square roots, cube roots and the smallest n

1. Prime factorisation

☐ I revised this

- Every whole number above 1 can be written as a product of primes in exactly one way.
- Use a factor tree or keep dividing by the smallest prime (2, 3, 5, 7, 11, ...).
- Write the answer in index form: $360 = 2^3 \times 3^2 \times 5$.

Repeated division

1. Divide by 2 until you can't.
2. Then by 3, then 5, then 7...
3. Stop at 1; collect equal primes as powers.

Worked example

Write 360 as a product of its prime factors.

1. $360 \div 2 = 180$, $\div 2 = 90$, $\div 2 = 45$.
2. $45 \div 3 = 15$, $\div 3 = 5$, $\div 5 = 1$.
3. Primes used: 2, 2, 2, 3, 3, 5.

$$2^3 \times 3^2 \times 5$$

2. HCF and LCM

☐ I revised this

- **HCF** (highest common factor): the biggest number that divides into both.
- **LCM** (lowest common multiple): the smallest number both divide into.
- From prime factors: HCF = primes they **share**, lowest power of each. LCM = **every** prime, highest power of each.
- A Venn diagram of the prime factors works too: HCF = the overlap; LCM = everything in the diagram.

Formulas

HCF	shared primes, lowest powers
LCM	all primes, highest powers
HCF $\times$ LCM = $a \times b$	for two numbers a and b

Worked example

Find the HCF and LCM of 84 and 120.

1. $84 = 2^2 \times 3 \times 7$; $120 = 2^3 \times 3 \times 5$.
2. HCF: $2^2 \times 3 = 12$.
3. LCM: $2^3 \times 3 \times 5 \times 7 = 840$.

$$\text{HCF } 12, \text{ LCM } 840$$

3. Word problems

☐ I revised this

- 'Together again', 'at the same time', 'smallest number of each' → **LCM**.
- 'Largest possible', 'cut into equal pieces', 'share into identical groups' → **HCF**.
- Algebraic HCF: HCF of the numbers $\times$ the lowest power of each shared letter. HCF of $12x^{2y}$ and $18xy^3$ is $6xy$.

Worked example

Two lighthouses flash every 12 s and every 20 s. They flash together at 9:00. When do they next flash together?

1. Together again → LCM.
2. $12 = 2^2 \times 3$, $20 = 2^2 \times 5$, $\text{LCM} = 2^2 \times 3 \times 5 = 60$.

60 s later, at 9:01

4. Square roots, cube roots and the smallest n

☐ I revised this

- A **square** number has every prime to an even power; a **cube** has every power a multiple of 3.
- $\sqrt{}$: halve every power. $\sqrt[3]{}$: divide every power by 3. $\sqrt{(2^2 \times 3^2 \times 7^2)} = 2 \times 3 \times 7 = 42$.
- Smallest number to multiply by to make a square: supply whatever makes every power even.

Worked example

Find the smallest whole number n such that $120n$ is a cube number.

1. $120 = 2^3 \times 3 \times 5$.
2. For a cube every power must be a multiple of 3.
3. Need $3^2 \times 5^2$ more.

$n = 225$

Common mistakes

Don't write	Write instead
$\times$ LCM of 18 and 24 = $18 \times 24 = 432$	$\checkmark$ That is a common multiple, but the lowest is 72
$\times$ Writing $2 \times 2 \times 2 \times 3 \times 3 \times 5$ when 'index form' is asked	$\checkmark$ Write $2^3 \times 3^2 \times 5$
$\times$ Including 1 in a prime factorisation	$\checkmark$ 1 is not prime — leave it out
$\times$ Taking the highest powers for the HCF	$\checkmark$ HCF uses the LOWEST powers of shared primes

In the exam

- Always check the HCF divides both numbers and both numbers divide the LCM.
- In word problems, say which one you are finding (HCF or LCM) before calculating.
- Show the prime factorisations — they earn method marks even if a multiplication slips.

My notes

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Graphs Of Functions

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Linear, quadratic, cubic, reciprocal and exponential graphs: shapes, tables, plotting and key features.

1. Shapes of graphs
2. Tables of values and plotting
3. Key features
4. Reading from a graph

1. Shapes of graphs

☐ I revised this

- Recognise each family from its equation and sketch its shape.

Type	Example	Shape and features
Linear	$y = 2x + 1$	straight line
Quadratic	$y = x^2 - 4$	U if x^2 term positive, $\cap$ if negative; symmetrical
Cubic	$y = x^3 - 3x$	S-shaped; up to two turning points
Reciprocal	$y = 2/x$	two branches; asymptotes $x = 0$ and $y = 0$
Exponential	$y = 2^x$	through $(0, 1)$; asymptote $y = 0$; grows fast

Worked example

Which type is $y = 5 - x^2$, and where is its turning point?

1. Quadratic with a negative x^2 term: $\cap$ -shaped.
2. At $x = 0$, $y = 5$.

$\cap$ -shaped parabola, maximum at $(0, 5)$

2. Tables of values and plotting

☐ I revised this

- Use brackets for negatives: $x = -2$ in $x^2 - 3x + 1$ gives $4 + 6 + 1 = 11$.
- Use the calculator's TABLE mode to check every value.
- Plot with small crosses, then draw **one smooth curve** through all points — no straight segments, no joining across an asymptote.

How to do it

1. Complete the table (check with TABLE mode).
2. Plot each point accurately.
3. Draw a smooth curve; extend only as far as the table.

Worked example

For $y = 12/x + x$, find y when $x = 4$ and $x = -2$.

1. $x = 4$: $3 + 4 = 7$.
2. $x = -2$: $-6 + (-2) = -8$.

7 and -8

3. Key features

☐ I revised this

- **y-intercept:** put $x = 0$. **Roots** (x-intercepts): solve $y = 0$.
- **Turning point** of a quadratic: halfway between the roots, or from completed-square form $(x - p)^2 + q \rightarrow (p, q)$.
- Cubic in factorised form $y = (x - a)(x - b)(x - c)$ crosses the x-axis at a , b and c .

Worked example

For $y = x^2 - 4x - 5$, find the intercepts and the turning point.

1. $x = 0$: $y = -5$.
2. $x^2 - 4x - 5 = (x - 5)(x + 1) = 0 \rightarrow x = 5$ or -1 .
3. Halfway: $x = 2$, $y = 4 - 8 - 5 = -9$.

(0, -5); (5, 0) and (-1, 0); minimum (2, -9)

4. Reading from a graph

☐ I revised this

- Read solutions of 'curve = 0' where the curve crosses the x-axis.
- Read a y value: go up from x to the curve, then across.
- Give values to the accuracy of the grid (often to the nearest half small square).

Worked example

The graph of $y = x^3 - 3x$ crosses the x-axis three times. Where?

1. $x(x^2 - 3) = 0$.
2. $x = 0$ or $x = \pm\sqrt{3} \approx \pm 1.73$.

$x = -1.7, 0$ and 1.7

Common mistakes

Don't write	Write instead
$\times$ $(-2)^2$ entered as $-2^2 = -4$	$\checkmark$ Use brackets: $(-2)^2 = 4$
$\times$ Joining points with straight lines	$\checkmark$ Draw one smooth curve
$\times$ Joining the two branches of $y = k/x$	$\checkmark$ They never meet — break the curve at $x = 0$
$\times$ Putting the turning point of $(x - 3)^2 + 4$ at $(-3, 4)$	$\checkmark$ It is at $(3, 4)$

In the exam

- Draw curves in pencil with one confident stroke.
- Use the symmetry of a quadratic to check your table.
- Label the curve with its equation when there is more than one on the grid.

My notes

Algebra

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Expressions, expanding, linear equations, forming equations, changing the subject, algebraic fractions and quadratics.

1. Expressions and substitution
2. Expanding brackets
3. Solving linear equations
4. Forming expressions and equations
5. Changing the subject
6. Algebraic fractions
7. Quadratic equations

1. Expressions and substitution

☐ I revised this

- Collect **like terms** only: $3x + 2y - x = 2x + 2y$; x^2 and x are not like terms.
- Substitute with brackets round negative numbers: if $a = -2$, then $a^2 = (-2)^2 = 4$ and $3a = -6$.
- Write terms in a standard way: $3 \times a \times a \times b = 3a^2 b$.

Worked example

Find the value of $2p^2 - 5q$ when $p = 4$ and $q = -3$.

1. $2 \times 4^2 = 2 \times 16 = 32$.
2. $5 \times (-3) = -15$.
3. $32 - (-15) = 32 + 15$.

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2. Expanding brackets

☐ I revised this

- Single bracket: multiply every term inside. $-3(2x - 5) = -6x + 15$ (watch the signs).
- Two brackets: every term in the first $\times$ every term in the second (FOIL or a grid), then collect like terms.
- Three brackets: expand two of them first, then multiply the result by the third.
- Perfect squares: $(a + b)^2 = a^2 + 2ab + b^2$ — never just $a^2 + b^2$.

Formulas

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Worked example

Expand and simplify $(x + 2)(x - 3)(2x + 1)$.

1. $(x + 2)(x - 3) = x^2 - x - 6$.
2. $(x^2 - x - 6)(2x + 1) = 2x^3 + x^2 - 2x^2 - x - 12x - 6$.
3. Collect like terms.

$2x^3 - x^2 - 13x - 6$

3. Solving linear equations

☐ I revised this

- Do the same to both sides until the letter is on its own.
- Brackets: expand first. Letters on both sides: collect them on the side with more.
- Fractions: multiply every term by the lowest common denominator first.
- Algebraic denominators: $3/(x - 1) = 2/(x + 2) \rightarrow$ cross-multiply: $3(x + 2) = 2(x - 1) \rightarrow x = -8$.

How to do it

1. Clear fractions ($\times$ LCD).
2. Expand brackets.
3. Collect x terms on one side, numbers on the other.
4. Divide by the number in front of x .
5. Check by substituting.

Worked example

Solve $(x + 1)/3 - (x - 2)/4 = 2$.

1. Multiply every term by 12: $4(x + 1) - 3(x - 2) = 24$.
2. $4x + 4 - 3x + 6 = 24$.
3. $x + 10 = 24$.

$x = 14$

Exam tip: Put the second numerator in brackets: $-3(x - 2) = -3x + 6$, not $-3x - 6$.

4. Forming expressions and equations

☐ I revised this

- Turn words into algebra: '3 more than x ' = $x + 3$, 'twice x ' = $2x$, ' x squared' = x^2 .
- Perimeter adds all the sides; area multiplies; angles in a triangle add to 180° .
- Define your letter clearly, form the equation, solve it, then answer the question asked.

Worked example

A rectangle's length is 3 cm more than its width. Its perimeter is 34 cm. Find its area.

1. Width w , length $w + 3$.
2. $2(w + w + 3) = 34 \rightarrow 4w + 6 = 34$.
3. $w = 7$, length 10.

70 cm^2

5. Changing the subject

☐ I revised this

- Undo the operations on the new subject in reverse order, doing the same to both sides.
- Square $\leftrightarrow$ square root; $\times \leftrightarrow \div$; $+$ $\leftrightarrow$ $-$.
- If the subject appears twice: collect those terms on one side, **factorise** it out, then divide.

When the subject appears twice

1. Clear fractions and roots.
2. Expand any brackets.
3. Get every term with the subject on one side.
4. Factorise the subject out.
5. Divide.

Worked example

Make t the subject of $3t + a = bt - 5$.

1. Collect t terms: $3t - bt = -5 - a$.
2. Factorise: $t(3 - b) = -5 - a$.
3. Divide by $(3 - b)$.

$$t = (a + 5)/(b - 3)$$

Exam tip: Example with a root: $V = \frac{1}{3}\pi r^2 h$ gives $r = \sqrt{(3V/(\pi h))}$.

6. Algebraic fractions

☐ I revised this

- **Simplify:** factorise top and bottom completely, then cancel whole brackets (never single terms).
- **Add/subtract:** common denominator (often the product of the bottoms), then simplify the top.
- **Multiply/divide:** like number fractions — flip the second when dividing; factorise and cancel first.

Worked example

Write $2/(x + 1) + 3/(x - 2)$ as a single fraction.

1. Common denominator $(x + 1)(x - 2)$.
2. Top: $2(x - 2) + 3(x + 1) = 2x - 4 + 3x + 3$.
3. $= 5x - 1$.

$$(5x - 1)/((x + 1)(x - 2))$$

Exam tip: You can cancel $(x + 3)$ from top and bottom, but not the x in $(x + 3)/x$.

7. Quadratic equations

☐ I revised this

- Rearrange to $ax^2 + bx + c = 0$ first.
- **Factorise** when it works; each bracket = 0 gives a solution.
- Use the **formula** when the question asks for decimals ('2 d.p.', '3 s.f.').
- **Completing the square:** $x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$. It gives the turning point and exact solutions.
- If $b^2 - 4ac$ is negative there are no real solutions.

Formulas

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$$

the quadratic formula

$$x^2 + bx + c = (x + b/2)^2 - (b/2)^2 + c$$

completing the square

Worked example

Solve $2x^2 - 5x - 1 = 0$, giving your answers to 2 decimal places.

1. $a = 2$, $b = -5$, $c = -1$.
2. $x = (5 \pm \sqrt{(25 + 8)}) \div 4 = (5 \pm \sqrt{33}) \div 4$.
3. $\sqrt{33} = 5.7446\dots$

$$x = 2.69 \text{ or } x = -0.19$$

Exam tip: Completing the square on $x^2 - 8x + 11$ gives $(x - 4)^2 - 5$: minimum point $(4, -5)$, and solutions $x = 4 \pm \sqrt{5}$.

Common mistakes

Don't write	Write instead
$\times (x + 3)^2 = x^2 + 9$	$\checkmark$ There is a middle term: $x^2 + 6x + 9$
$\times -3(x - 2) = -3x - 6$	$\checkmark$ Negative times negative: $-3x + 6$
$\times x(x + 3) = 40$, so $x = 40$ or $x + 3 = 40$	$\checkmark$ Only '= 0' lets you split: $x^2 + 3x - 40 = 0$
$\times$ Cancelling x in $(x + 3)/x$ to get 3	$\checkmark$ Only whole factors cancel
$\times$ Dividing only part of the top by $2a$ in the formula	$\checkmark$ Divide the whole of $-b \pm \sqrt{b^2 - 4ac}$ by $2a$

In the exam

- Show the substitution into the quadratic formula — it is worth a mark on its own.
- Give both solutions of a quadratic unless the context rules one out (a negative length).
- Check a linear equation's answer by substituting it back in.

My notes

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Sets

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Set language and symbols, Venn diagrams with two or three sets, shading, and word problems.

1. Set notation
2. Venn diagrams and shading
3. Venn diagram problems

1. Set notation

☐ I revised this

- Learn the symbols — questions use them without explanation.

Symbol	Meaning
ξ	the universal set (everything)
$x \in A$	x is in A
$x \notin A$	x is not in A
A'	everything not in A
$A \cup B$	in A or B or both (union)
$A \cap B$	in both A and B (intersection)
$\emptyset$	the empty set
$A \subseteq B$	A is a subset of B
$n(A)$	the number of elements in A
$\{x : x > 3\}$	the set of x such that $x > 3$

Worked example

$A = \{2, 4, 6, 8, 10\}$, $B = \{3, 6, 9\}$. Find $A \cap B$ and $n(A \cup B)$.

1. Both: 6.
2. $A \cup B = \{2, 3, 4, 6, 8, 9, 10\}$.

$A \cap B = \{6\}$; $n(A \cup B) = 7$

2. Venn diagrams and shading

☐ I revised this

- Two sets make four regions: A only, both, B only, neither.
- $A \cap B' =$ in A but not B . $(A \cup B)' =$ outside both. $A' \cup B =$ everything except ' A only'.
- For three sets, there are eight regions; fill from the centre outwards.

Region	Notation
A only	$A \cap B'$
B only	$A' \cap B$
Both	$A \cap B$
Neither	$(A \cup B)'$ or $A' \cap B'$
Exactly one	$(A \cap B') \cup (A' \cap B)$

Worked example

Describe the region 'in B but not in A or C' using set notation.

1. In B, not in A, not in C.

$$B \cap A' \cap C'$$

3. Venn diagram problems

☐ I revised this

- Start with the overlap, then subtract it from each set's total.
- Remember the region outside the circles.
- With an unknown, write each region in terms of x and set the total equal to $n(\xi)$.
- Probability from a Venn diagram = number in the region $\div$ the total (or $\div$ the given set for 'given that').

Formulas

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Worked example

In a group of 50, 32 like tea, 25 like coffee and 6 like neither. How many like only tea?

1. Let x like both: $(32 - x) + x + (25 - x) + 6 = 50$.
2. $63 - x = 50$, $x = 13$.
3. Only tea = $32 - 13$.

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Common mistakes

Don't write	Write instead
$\times$ $n(A \cup B) = n(A) + n(B)$	$\checkmark$ Subtract the overlap
$\times$ Putting 32 in the 'tea only' region	$\checkmark$ 32 is all of tea; tea only = $32 - \text{both}$
$\times$ Forgetting the region outside the circles	$\checkmark$ It is part of ξ — include it in the total
$\times$ Listing 1 as a prime in a set	$\checkmark$ 1 is not prime

In the exam

- Draw the Venn diagram even when not asked — it organises the information.
- Check that all the regions add up to $n(\xi)$.
- Shading questions: shade carefully and clearly; mark the region boundaries.

My notes

Probability

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Probability scale, single events, relative and expected frequency, sample spaces, tree diagrams and conditional probability.

1. Single events
2. Relative and expected frequency
3. Sample spaces and combined events
4. Tree diagrams
5. Conditional probability

1. Single events

☐ I revised this

- Probability is from 0 (impossible) to 1 (certain). Write it as a fraction, decimal or percentage.
- $P(\text{event}) = \text{number of ways it happens} \div \text{total number of equally likely outcomes}$.
- $P(\text{not } A) = 1 - P(A)$. The probabilities of all possible outcomes add to 1.

Formulas

$$P(A) = \text{favourable} \div \text{total}$$

$$P(\text{not } A) = 1 - P(A)$$

Worked example

The only outcomes A, B and C have probabilities 0.2, 0.45 and x. Find x.

$$1. \ 0.2 + 0.45 + x = 1.$$

$$x = 0.35$$

2. Relative and expected frequency

☐ I revised this

- Relative frequency = number of times it happened $\div$ number of trials. It estimates the probability; more trials = better estimate.
- Expected frequency = probability $\times$ number of trials.

Worked example

A spinner landed on 6 in 45 of 300 spins. How many 6s would you expect in 1000 spins?

$$1. \ \text{Relative frequency} = 45/300 = 0.15.$$

$$2. \ 0.15 \times 1000.$$

$$150$$

3. Sample spaces and combined events

☐ I revised this

- A sample space diagram (grid) lists every outcome of two events — useful for two dice or two spinners.
- **AND** (independent events): multiply. **OR** (events that can't both happen): add.
- 'At least one' = $1 - P(\text{none})$.

Formulas

$$P(A \text{ and } B) = P(A) \times P(B)$$

independent

$$P(A \text{ or } B) = P(A) + P(B)$$

mutually exclusive

$$P(\text{at least one}) = 1 - P(\text{none})$$

Worked example

Two fair dice are thrown. Find the probability that the total is 7.

1. 36 outcomes in the grid.
2. Totals of 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) — 6 outcomes.

$$6/36 = 1/6$$

4. Tree diagrams

☐ I revised this

- Each set of branches adds to 1.
- Multiply **along** a path; add the paths you want.
- **With replacement**: the second-stage probabilities are the same as the first.
- **Without replacement**: one fewer item in total, and one fewer of the colour already taken.

How to do it

1. Draw the branches with probabilities.
2. Multiply along each wanted path.
3. Add the results of all wanted paths.

Worked example

A bag has 4 red and 6 blue sweets. Two are eaten without replacement. Find $P(\text{one of each colour})$.

1. $P(R \text{ then } B) = 4/10 \times 6/9 = 24/90$.
2. $P(B \text{ then } R) = 6/10 \times 4/9 = 24/90$.
3. Add: $48/90$.

$$8/15$$

5. Conditional probability

☐ I revised this

- 'Given that' narrows the total to the group you are told about.
- From a table or Venn diagram: count only within the given group.
- From a tree: the second branches are already conditional on the first.

Formulas

$$P(A \text{ given } B) = P(A \text{ and } B) \div P(B)$$

Worked example

12 of 20 girls and 6 of 10 boys wear glasses. A student who wears glasses is chosen. Find the probability it is a boy.

1. 18 students wear glasses.
2. 6 of them are boys.

$$6/18 = 1/3$$

Common mistakes

Don't write	Write instead
✗ Adding along the branches	✓ Multiply along a path; add the results
✗ Same denominator on both stages without replacement	✓ 4/10 then 3/9 (or 6/9)
✗ A probability bigger than 1	✓ Probabilities are between 0 and 1
✗ Writing a probability as '3 out of 5' or 3 : 5	✓ Write 3/5, 0.6 or 60%
✗ Forgetting one of the two orders for 'one of each'	✓ Red-then-blue AND blue-then-red

In the exam

- Draw the tree or grid even when not asked.
- Fractions don't need simplifying unless asked — but keep them exact.
- Check: all end probabilities of a tree add to 1.

My notes

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Vectors

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Column vectors, magnitude, position vectors, routes in diagrams, ratios, parallel vectors and collinear points.

1. Column vectors
2. Magnitude
3. Position vectors
4. Vectors in diagrams

1. Column vectors

☐ I revised this

- A column vector has a top number (x, across) and a bottom number (y, up).
- Add or subtract top with top and bottom with bottom.
- Multiply by a number: multiply both parts. $3 \times (-2, 4) = (-6, 12)$.
- Written $\begin{pmatrix} a \\ b \end{pmatrix}$ (by hand) or $\overrightarrow{AB}$ with an arrow.

Worked example

$a = (3, -2)$ and $b = (-1, 5)$. Find $2a - b$.

1. $2a = (6, -4)$.
 2. $(6 - (-1), -4 - 5)$.
- $(7, -9)$**

2. Magnitude

☐ I revised this

- The magnitude $|a|$ is the length of the vector: Pythagoras on its two parts.

Formulas

$$|(x, y)| = \sqrt{x^2 + y^2}$$

Worked example

Find the magnitude of $(5, -12)$.

1. $\sqrt{(25 + 144)} = \sqrt{169}$.

13

3. Position vectors

☐ I revised this

- The position vector of A is $\overrightarrow{OA}$ (from the origin).
- $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$.
- Midpoint M of AB has position vector $\frac{1}{2}(a + b)$.

Formulas

$$\overrightarrow{AB} = b - a$$

end minus start

$$\overrightarrow{OM} = \frac{1}{2}(a + b)$$

midpoint

Transformations

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Reflections, rotations, translations and enlargements: drawing, describing and combining them.

1. Reflection
2. Rotation
3. Translation
4. Enlargement
5. Describing and combining

1. Reflection

☐ I revised this

- Every point moves to the same distance on the other side of the mirror line.
- Describe it with the **equation of the mirror line**: $x = 2$, $y = -1$, $y = x$, $y = -x$.
- In the x-axis: $(x, y) \rightarrow (x, -y)$. In the y-axis: $(x, y) \rightarrow (-x, y)$. In $y = x$: $(x, y) \rightarrow (y, x)$.

Worked example

Reflect the point (2, 7) in the line $y = x$.

1. Swap the coordinates.

(7, 2)

2. Rotation

☐ I revised this

- Describe with **centre**, **angle** and **direction** (180° needs no direction).
- Use tracing paper: hold the centre with your pencil and turn.
- About the origin: 90° anticlockwise $(x, y) \rightarrow (-y, x)$; 90° clockwise $(x, y) \rightarrow (y, -x)$; 180° $(x, y) \rightarrow (-x, -y)$.

Worked example

Rotate (4, 1) through 90° anticlockwise about the origin.

1. $(x, y) \rightarrow (-y, x)$.

(-1, 4)

3. Translation

☐ I revised this

- Every point moves the same distance in the same direction.
- Describe with a **column vector**: top number = right (+) / left (-); bottom number = up (+) / down (-).

Worked example

Translate $(-2, 3)$ by the vector with top 5 and bottom -4 .

1. $x: -2 + 5 = 3$.

2. $y: 3 - 4 = -1$.

(3, -1)

4. Enlargement

☐ I revised this

- Describe with **centre** and **scale factor**.
- Distance from the centre $\times$ scale factor gives the new position.
- Scale factor between 0 and 1 makes it smaller; negative puts the image on the other side of the centre, upside down.
- Find the centre: join matching corners of the shape and image and extend the lines to where they meet.

How to do it

1. Find the vector from the centre to a corner.
2. Multiply it by the scale factor.
3. Add the result to the centre.
4. Repeat for each corner.

Worked example

Triangle A has vertices (1, 1), (2, 1), (1, 3). Triangle B has (3, 3), (5, 3), (3, 7). Describe the transformation.

1. Lengths double: scale factor 2.
2. Joining matching corners meets at $(-1, -1)$.

Enlargement, scale factor 2, centre $(-1, -1)$

5. Describing and combining

☐ I revised this

- Name only **one** single transformation when asked — two answers score nothing.
- Use the exact words: reflection, rotation, translation, enlargement.
- Combinations: do the first, then the second, then describe the single transformation that does both.
- Reflection in the x-axis then the y-axis = rotation of 180° about the origin.

Transformation	You must give
Reflection	mirror line equation
Rotation	centre, angle, direction
Translation	column vector
Enlargement	centre, scale factor

Worked example

Reflect in $x = 1$, then in $x = 4$. What single transformation is this?

1. A point at $x = 0 \rightarrow x = 2 \rightarrow x = 6$: it moved 6 right; every point moves the same.

Translation by the vector with top 6 and bottom 0

Common mistakes

Don't write	Write instead
✗ 'Turn' or 'move' as the name	✓ Use rotation, translation, reflection, enlargement
✗ Reflecting in $y = x$ by changing the signs	✓ Swap the coordinates
✗ Enlarging from the origin when the centre is elsewhere	✓ Measure every distance from the given centre
✗ Giving two transformations when asked for one	✓ Find the single transformation

In the exam

- Check your image with tracing paper.
- Label images as instructed (A, B, C) — unlabelled shapes can lose marks.
- For enlargements, check the image size: every length $\times$ scale factor.

My notes

Similarity

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Similar shapes, scale factors for lengths, areas and volumes, and nested triangles.

1. Similar shapes and lengths
2. Nested triangles
3. Areas and volumes of similar shapes

1. Similar shapes and lengths

☐ I revised this

- Similar shapes have equal angles and sides in the same ratio.
- Length scale factor $k = \text{a length on the new shape} \div \text{the matching length on the old one}$.
- Two triangles are similar if two pairs of angles are equal (the third then matches too).

How to do it

1. Identify matching sides (opposite equal angles).
2. Find k from a pair of known matching sides.
3. Multiply (or divide) to find the missing length.

Worked example

Two similar triangles have matching sides 4 cm and 10 cm. A side of 6 cm on the small one matches which length?

1. $k = 10 \div 4 = 2.5$.
2. 6×2.5 .

15 cm

2. Nested triangles

☐ I revised this

- A line parallel to one side of a triangle cuts off a smaller similar triangle.
- Use the **whole** sides of the big triangle, not just the extra piece.
- Redraw the two triangles separately the same way round.

Worked example

In triangle ABC, $DE \parallel BC$ with D on AB, E on AC. $AD = 3$, $DB = 6$, $DE = 4$. Find BC.

1. $AB = 3 + 6 = 9$.
2. $k = 9 \div 3 = 3$.
3. $BC = 4 \times 3$.

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3. Areas and volumes of similar shapes

☐ I revised this

- Length factor $k \rightarrow$ area factor $k^2 \rightarrow$ volume factor k^3 .
- Surface area scales like area (k^2); mass and capacity scale like volume (k^3).
- Work backwards with roots: area ratio $\rightarrow k = \sqrt{\text{area ratio}}$; volume ratio $\rightarrow k = \sqrt[3]{\text{volume ratio}}$.

Formulas

$$\text{area factor} = k^2$$

$$\text{volume factor} = k^3$$

$$k = \sqrt{\text{area ratio}} = \sqrt[3]{\text{volume ratio}}$$

Worked example

Two similar jugs have surface areas 180 cm^2 and 405 cm^2 . The small one holds 0.8 litres. How much does the large one hold?

1. Area factor = $405 \div 180 = 2.25$, so $k = 1.5$.

2. Volume factor = $1.5^3 = 3.375$.

3. 0.8×3.375 .

2.7 litres

Common mistakes

Don't write	Write instead
✗ Area doubles when the lengths double	✓ Area $\times 4$ ($k^2 = 2^2$)
✗ Using DB instead of AB for the big triangle	✓ Use the whole side AB
✗ Cubing the volume ratio to get a length	✓ Take the cube root
✗ Showing similarity with one pair of equal angles	✓ You need two pairs of equal angles

In the exam

- Write k clearly as a fraction or decimal before using it.
- For area or volume questions, find the length factor first.
- To prove similarity, state the equal angles with reasons.

My notes

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Congruent

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Congruent shapes, the four triangle conditions and using them to find lengths and angles.

1. What congruent means
2. The four conditions
3. Using congruence

1. What congruent means

☐ I revised this

- Congruent shapes are **identical in shape and size**. They may be rotated or reflected.
- Corresponding sides are equal and corresponding angles are equal.
- Similar shapes have equal angles but sides in proportion — different sizes are allowed.

Worked example

Two triangles have the same three angles but different sides. Are they congruent?

1. Same angles only shows they are similar.

No — they are similar, not necessarily congruent

2. The four conditions

☐ I revised this

- Two triangles are congruent if they match on one of these:

Condition	What must be equal
SSS	all three sides
SAS	two sides and the angle BETWEEN them
ASA (or AAS)	two angles and a corresponding side
RHS	right angle, hypotenuse and one other side

Worked example

Triangles have $AB = DE$, angle $B = \text{angle } E$ and $BC = EF$. Are they congruent? Which condition?

1. Two sides equal; the equal angle is between them.

Yes, SAS

3. Using congruence

☐ I revised this

- Match the vertices in order: $ABC \cong PQR$ means $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$.
- Then the side $AB = PQ$, $BC = QR$, angle $B = \text{angle } Q$, and so on.
- AAA and SSA do **not** prove congruence.

Worked example

$ABC \cong PQR$. $AB = 7 \text{ cm}$, $BC = 9 \text{ cm}$, angle $B = 48^\circ$. Find QR and angle Q .

1. $BC \leftrightarrow QR$.
2. Angle $B \leftrightarrow \text{angle } Q$.

$QR = 9 \text{ cm}$, angle $Q = 48^\circ$

Common mistakes

Don't write	Write instead
✗ Using AAA to show congruence	✓ AAA shows similarity only
✗ Matching sides by their position on the page	✓ Match by letter order or equal angles
✗ Calling any two sides and an angle SAS	✓ The angle must be between the two sides

In the exam

- Name the condition (SSS, SAS, ASA, RHS) in your answer.
- Redraw the triangles the same way round if they are turned or flipped.
- Mark equal sides and angles on the diagram first.

My notes

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Surds

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Exact roots: simplifying, calculating with surds, and rationalising denominators.

1. Simplifying surds
2. Adding, multiplying and expanding
3. Rationalising the denominator

1. Simplifying surds

☐ I revised this

- A **surd** is a root that is irrational, like $\sqrt{2}$ or $\sqrt{12}$. It is exact — keep it as a surd unless told to round.
- Find the **largest square factor**: $\sqrt{72} = \sqrt{(36 \times 2)} = 6\sqrt{2}$.
- Square factors to look for: 4, 9, 16, 25, 36, 49, 64, 81, 100.

Formulas

$$\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$$

$$\sqrt{a} \times \sqrt{a} = a$$

Worked example

Simplify $\sqrt{75} + \sqrt{12}$.

1. $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$.
2. $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$.
3. $5\sqrt{3} + 2\sqrt{3}$.

$7\sqrt{3}$

2. Adding, multiplying and expanding

☐ I revised this

- Only **like surds** add or subtract: $6\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$, but $\sqrt{2} + \sqrt{3}$ cannot be combined.
- Multiply: $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$; $2\sqrt{5} \times 3\sqrt{2} = 6\sqrt{10}$.
- Expand brackets just as in algebra, then simplify each term.
- $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$: the surds cancel. This is used to rationalise.

Worked example

Expand and simplify $(3 + \sqrt{2})^2$.

1. $(3 + \sqrt{2})(3 + \sqrt{2}) = 9 + 3\sqrt{2} + 3\sqrt{2} + 2$.
2. Collect: $11 + 6\sqrt{2}$.

$11 + 6\sqrt{2}$

Exam tip: $(a + \sqrt{b})^2$ is NOT $a^2 + b$ — there is a middle term $2a\sqrt{b}$.

3. Rationalising the denominator

☐ I revised this

- 'Rationalise' means: no surd on the bottom of the fraction.
- $a/\sqrt{b}$: multiply top and bottom by $\sqrt{b}$. $10/\sqrt{5} = 10\sqrt{5}/5 = 2\sqrt{5}$.
- $a/(b + \sqrt{c})$: multiply top and bottom by $(b - \sqrt{c})$. $a/(b - \sqrt{c})$: multiply by $(b + \sqrt{c})$.

How to do it

1. Look at the bottom.
2. Single surd $\sqrt{b} \rightarrow$ multiply top and bottom by $\sqrt{b}$.
3. $b \pm \sqrt{c} \rightarrow$ multiply top and bottom by the same with the opposite sign.
4. Expand and simplify; cancel any common factor.

Worked example

Rationalise $6/(3 - \sqrt{3})$.

1. Multiply by $(3 + \sqrt{3})/(3 + \sqrt{3})$.
2. Bottom: $9 - 3 = 6$.
3. Top: $6(3 + \sqrt{3})$.
4. $6(3 + \sqrt{3})/6$.

$3 + \sqrt{3}$

Common mistakes

Don't write	Write instead
$\times \sqrt{9} + \sqrt{16} = \sqrt{25}$	$\checkmark \sqrt{9} + \sqrt{16} = 3 + 4 = 7$
$\times$ Leaving $\sqrt{72}$ as $2\sqrt{18}$	$\checkmark$ Use the largest square factor: $6\sqrt{2}$
$\times (3 + \sqrt{2})^2 = 9 + 2$	$\checkmark$ Middle term: $9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$
$\times$ Multiplying only the top when rationalising	$\checkmark$ Multiply top and bottom by the same surd

In the exam

- 'Give your answer in the form $a + b\sqrt{c}$ ' — make sure a , b , c are integers and the surd is simplified.
- Surd questions are on the non-calculator paper; check by squaring your answer mentally.
- Exact answers (like in Pythagoras) are often wanted as simplified surds.

My notes

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3D Trigonometry

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Pythagoras and trigonometry in cuboids, pyramids and prisms, and the angle between a line and a plane.

1. Lengths in 3D
2. The angle between a line and a plane
3. Pyramids
4. Prisms and wedges

1. Lengths in 3D

☐ I revised this

- Find a right-angled triangle that contains the length you want, then use Pythagoras.
- Usually two steps: the diagonal of the base first, then the vertical triangle.
- Space diagonal of a cuboid $a \times b \times c$: $\sqrt{a^2 + b^2 + c^2}$.

Formulas

$$d = \sqrt{a^2 + b^2 + c^2}$$

space diagonal of a cuboid

Worked example

A box is 12 cm long, 4 cm wide and 3 cm high. Find the length of the longest rod that fits inside.

1. Base diagonal: $\sqrt{144 + 16} = \sqrt{160}$.
2. Space diagonal: $\sqrt{160 + 9} = \sqrt{169}$.

13 cm

2. The angle between a line and a plane

☐ I revised this

- Drop a perpendicular from the top of the line to the plane; the foot gives the line's **projection**.
- The angle is between the line and its projection — not the vertical.
- The right-angled triangle is: line (hypotenuse), projection (adjacent), vertical height (opposite).

How to do it

1. Identify the line and the plane.
2. Find the projection of the line on the plane.
3. Draw the right-angled triangle flat on its own.
4. Use SOH CAH TOA.

Worked example

A cuboid is 6 cm by 8 cm by 5 cm high. Find the angle between the space diagonal and the base.

1. Base diagonal (projection) = 10 cm.
2. $\tan \theta = 5 \div 10$.

26.6°

3. Pyramids

☐ I revised this

- The vertical height goes from the apex to the centre of the base.
- For a square or rectangular base, the centre is the midpoint of the base diagonal.
- The angle between a sloping edge and the base uses half the base diagonal; the angle between a sloping face and the base uses half the base width.

Worked example

A square-based pyramid has base $8\text{ cm} \times 8\text{ cm}$ and vertical height 6 cm . Find the angle between a sloping edge and the base.

1. Half the base diagonal $= 4\sqrt{2} = 5.657\text{ cm}$.
2. $\tan \theta = 6 \div 5.657$.

46.7°

4. Prisms and wedges

☐ I revised this

- Look for the right angles between the faces.
- Redraw each triangle you need as a flat 2D triangle with its lengths.
- You may need the sine or cosine rule in a non-right-angled face.

Worked example

A wedge has a horizontal base 12 cm by 5 cm and a vertical back edge 9 cm high above one 5 cm side. Find the length from a front bottom corner to the opposite top corner.

1. Base diagonal: $\sqrt{(144 + 25)} = 13$.
2. Then with the 9 cm height: $\sqrt{(169 + 81)} = \sqrt{250}$.

15.8 cm (3 s.f.)

Common mistakes

Don't write	Write instead
 Using the base side instead of the base diagonal	 The projection of a space diagonal is the base diagonal
 Measuring the angle from the vertical	 The angle with a plane is measured from the plane
 Rounding $\sqrt{160}$ to 12.6 early	 Keep it exact or keep all digits

In the exam

- Draw every triangle you use separately and label it.
- Name the angle with letters (e.g. angle AGE) so the examiner can follow.
- Give angles to 1 decimal place.

My notes

Exponential Growth And Decay

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Repeated percentage change: growth, decay, working backwards, finding the rate and the time.

1. Growth and decay multipliers
2. Working backwards and finding the rate
3. Finding the number of periods

1. Growth and decay multipliers

☐ I revised this

- Repeated percentage change multiplies by the same number each period.
- Growth of $r\%$ → multiplier $(1 + r/100)$. Decay of $r\%$ → multiplier $(1 - r/100)$.
- After n periods: amount = starting amount $\times$ multiplier ^{n} .
- Typical contexts: population, bacteria, compound interest (growth); depreciation, radioactive decay, cooling (decay).

Change	Multiplier
+4%	1.04
+12.5%	1.125
-15%	0.85
-2.5%	0.975
doubles	2
halves	0.5

Formulas

$A = P(1 + r/100)^n$	growth
$A = P(1 - r/100)^n$	decay

Worked example

A car worth \$12 000 loses 15% of its value each year. Find its value after 3 years.

1. Multiplier 0.85.
2. $12\,000 \times 0.85^3 = 7369.5$.

\$7369.50

2. Working backwards and finding the rate

☐ I revised this

- Starting amount = final amount $\div$ multiplier ^{n} .
- To find the rate: (final $\div$ start) = multiplier ^{n} , so take the n -th root to get the multiplier.
- Multiplier 1.1 means 10% growth; 0.92 means 8% decay.

Worked example

A value grows from 400 to 484 in 2 years at a constant percentage rate. Find the rate.

1. $484 \div 400 = 1.21$.
2. Multiplier = $\sqrt{1.21} = 1.1$.

10% per year

3. Finding the number of periods

☐ I revised this

- No logarithms needed: try $n = 1, 2, 3, \dots$ on the calculator until the amount passes the target.
- Give the first whole period that passes it, and show the values either side.
- Half-life questions: each half-life multiplies the amount by 0.5.

How to do it

1. Write amount = $P \times \text{multiplier}^n$.
2. Try values of n .
3. Write the value just below and just above the target.
4. Answer with the first n that passes it.

Worked example

\$800 is invested at 3% compound interest. After how many whole years is it first worth more than \$1000?

1. $800 \times 1.03^7 = 983.9\dots$

2. $800 \times 1.03^8 = 1013.4\dots$

8 years

Common mistakes

Don't write	Write instead
 15% depreciation for 2 years = 30% off	 Multiply by 0.85 twice: 0.7225 (27.75% off)
 Using 1.15 for a 15% decrease	 Decrease: 0.85
 Giving the year just before the target is reached	 Give the first whole period that passes it
 Dividing by 2 for the rate from a 2-year change	 Take the square root of the overall multiplier

In the exam

- Write the multiplier first; most method marks come from it.
- Show the trial values when finding the number of years.
- Read whether the question wants the amount, the increase/decrease, or the percentage.

My notes

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Constructions And Scale Drawings

Name: _____

Quick revision notes [Open it online](#)

Date: _____

At a glance

Measuring, ruler-and-compasses constructions, nets, scale drawings and three-figure bearings.

1. Measuring and drawing
2. Triangles from three sides
3. Bisectors
4. Nets
5. Scale drawings and bearings

1. Measuring and drawing

☐ I revised this

- Measure lengths to the nearest millimetre and angles to the nearest degree.
- Use the correct scale on the protractor (inner or outer) — start from the 0 on the arm you measure from.
- Leave all construction lines and arcs showing.

Worked example

A line measures 7.4 cm on a drawing with scale 1 cm to 5 m. How long is it in real life?

1. 7.4×5 .

37 m

2. Triangles from three sides

☐ I revised this

- Only a ruler and compasses are allowed.

How to do it

1. Draw the longest side as the base.
2. Set the compasses to the second length; draw an arc from one end.
3. Set them to the third length; draw an arc from the other end.
4. Join the crossing point to both ends.

Worked example

Construct a triangle with sides 7 cm, 5 cm and 6 cm. What do you draw after the 7 cm base?

1. Arc of radius 5 cm from one end.
2. Arc of radius 6 cm from the other end.

Join the point where the arcs cross to both ends

3. Bisectors

☐ I revised this

- **Perpendicular bisector** of AB: arcs of the same radius (more than half AB) from A and from B, above and below; join the crossings. Every point on it is equally far from A and B.
- **Angle bisector**: arc from the vertex cutting both arms; equal arcs from those two points; join the vertex to the crossing. Every point on it is equally far from both arms.

Worked example

Which construction finds the points equally distant from two towns A and B?

1. Equal distance from two points.

The perpendicular bisector of AB

4. Nets

☐ I revised this

- A net is the flat shape that folds into a solid.
- Cube: 6 squares. Cuboid: 3 pairs of rectangles. Triangular prism: 2 triangles + 3 rectangles.
- Square-based pyramid: 1 square + 4 triangles. Cylinder: 1 rectangle + 2 circles.
- The surface area is the area of the net.

Worked example

How many faces does the net of a triangular prism have?

1. 2 triangles and 3 rectangles.

5

5. Scale drawings and bearings

☐ I revised this

- Scale 1 : n: real length = drawing length $\times$ n (watch the units).
- Bearings: measured **from north, clockwise**, written with three figures (045°, 120°, 300°).
- 'Bearing of B from A': stand at A, face north, turn clockwise to B.
- Back bearing (A from B) = bearing $\pm$ 180°.

Formulas

back bearing = bearing $\pm$ 180°

add if less than 180°, subtract if more

Worked example

The bearing of B from A is 070°. Find the bearing of A from B.

1. 070 + 180.

250°

Exam tip: Always draw a north line at the point you measure FROM.

Common mistakes

Don't write	Write instead
✗ Writing a bearing as 45°	✓ Three figures: 045°
✗ Bearing of A from B = bearing of B from A	✓ Add or subtract 180°
✗ Rubbing out construction arcs	✓ Leave them — they earn the marks
✗ Using a protractor in a 'compasses only' construction	✓ Only arcs and a straight edge

In the exam

- Use a sharp pencil; accuracy marks allow about ± 2 mm and $\pm 2^\circ$.
- Check the scale units (cm to km?) before converting.
- For bearings, sketch the diagram with north lines before calculating.

My notes

Trigonometric Graphs

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Exact trig values, the graphs of $y = \sin x$, $\cos x$ and $\tan x$, and solving trig equations from 0° to 360° .

1. Exact values
2. The graphs of $\sin$, $\cos$ and $\tan$
3. Solving trig equations

1. Exact values

☐ I revised this

- Learn the table — it is needed on the non-calculator paper.
- Memory aid: $\sin$ goes 0, $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{2}$, 1 from 0° to 90° ; $\cos$ is the same list backwards.

	0°	30°	45°	60°	90°
$\sin$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Worked example

Work out $4 \sin 30^\circ + \tan 45^\circ$.

1. $\sin 30^\circ = \frac{1}{2}$ and $\tan 45^\circ = 1$.
2. $4 \times \frac{1}{2} + 1$.

3

2. The graphs of $\sin$, $\cos$ and $\tan$

☐ I revised this

- $y = \sin x$: starts at 0, maximum 1 at 90° , 0 at 180° , minimum -1 at 270° , 0 at 360° .
- $y = \cos x$: starts at 1, 0 at 90° , -1 at 180° , 0 at 270° , 1 at 360° .
- $y = \tan x$: through $(0, 0)$, $(180^\circ, 0)$, $(360^\circ, 0)$; asymptotes at $x = 90^\circ$ and 270° ; repeats every 180° .
- $y = a \sin x$ stretches it vertically: maximum a , minimum $-a$.

x	0°	90°	180°	270°	360°
$\sin x$	0	1	0	-1	0
$\cos x$	1	0	-1	0	1
$\tan x$	0	—	0	—	0

Worked example

State the maximum and minimum values of $y = 3 \cos x$ and where the maximum occurs ($0^\circ \leq x \leq 360^\circ$).

1. $\cos x$ is between -1 and 1 .

Maximum 3 at $x = 0^\circ$ and 360° ; minimum -3

3. Solving trig equations

☐ I revised this

- Rearrange to $\sin x = k$, $\cos x = k$ or $\tan x = k$ first.
- The calculator gives one angle; use the graph's symmetry for the others in 0° to 360° .

How to do it

1. Rearrange to trig function = number.
2. Use the inverse on the calculator to get x_1 .
3. Find the second solution from the table (add 360° to any negative angle first).
4. Check both are in the range.

Equation	Solutions in 0° – 360° (x_1 from the calculator)
$\sin x = k$	x_1 and $180^\circ - x_1$
$\cos x = k$	x_1 and $360^\circ - x_1$
$\tan x = k$	x_1 and $x_1 + 180^\circ$

Worked example

Solve $2 \sin x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$.

1. $\sin x = -0.5$.
2. Calculator: $x = -30^\circ$, outside the range.
3. Solutions: $180^\circ + 30^\circ = 210^\circ$ and $360^\circ - 30^\circ = 330^\circ$.

$x = 210^\circ$ or $x = 330^\circ$

Exam tip: Sketch the graph with the horizontal line $y = k$ to see how many solutions there are.

Common mistakes

Don't write	Write instead
✗ Giving only the calculator's answer	✓ There are usually two answers in 0° – 360°
✗ $\sin x = -0.5 \rightarrow x = -30^\circ$	✓ Negative angles are outside the range: use 210° and 330°
✗ Calculator in radians	✓ Set it to degrees
✗ $\tan x = k \rightarrow x_1$ and $180^\circ - x_1$	✓ $\tan$ repeats every 180° : x_1 and $x_1 + 180^\circ$

In the exam

- Learn the exact values — they appear on the non-calculator paper.
- Sketch the graph to check your second solution.
- Give angles to 1 decimal place unless they are exact.

My notes

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Differentiation

Quick revision notes [Open it online](#)

Name: _____

Date: _____

At a glance

Differentiating ax^n , gradients and tangents, and turning points with maximum or minimum.

1. Differentiating
2. Gradients and tangents
3. Turning points

1. Differentiating

☐ I revised this

- The derivative dy/dx gives the **gradient** of the curve at any point.
- For ax^n : multiply by the power, then take one off the power.
- A term in x differentiates to a number ($7x \rightarrow 7$); a constant differentiates to 0.
- Differentiate a sum term by term.

Formulas

$$y = ax^n \rightarrow dy/dx = nax^{n-1}$$

$$y = kx \rightarrow dy/dx = k$$

$$y = c \rightarrow dy/dx = 0$$

Worked example

Differentiate $y = x^4 - 3x^2 + 7x - 2$.

1. $x^4 \rightarrow 4x^3$.

2. $-3x^2 \rightarrow -6x$.

3. $7x \rightarrow 7$; $-2 \rightarrow 0$.

$$dy/dx = 4x^3 - 6x + 7$$

2. Gradients and tangents

☐ I revised this

- Gradient at a point: differentiate, then substitute the x value.
- Where is the gradient k ? Set $dy/dx = k$ and solve.
- Equation of the tangent at (x_1, y_1) : $m = dy/dx$ there, then $y - y_1 = m(x - x_1)$.

Formulas

$$y - y_1 = m(x - x_1)$$

tangent through (x_1, y_1)

Worked example

Find the equation of the tangent to $y = x^2 + 3x$ at $x = 1$.

1. $y = 1 + 3 = 4$ at $x = 1$.

2. $dy/dx = 2x + 3 = 5$ at $x = 1$.

3. $y - 4 = 5(x - 1)$.

$$y = 5x - 1$$

3. Turning points

☐ I revised this

- At a turning (stationary) point the gradient is 0: solve $dy/dx = 0$.
- Find y by substituting x into the **original** equation.
- Maximum or minimum? Second derivative d^2y/dx^2 : negative $\rightarrow$ maximum, positive $\rightarrow$ minimum. Or check the gradient just either side.
- Real-life problems: maximise area, profit, volume by finding where the derivative is 0.

How to do it

1. Differentiate.
2. Set $dy/dx = 0$ and solve for x .
3. Substitute into y for the coordinates.
4. Use d^2y/dx^2 (or the gradient either side) to decide max or min.

Worked example

Find the turning points of $y = x^3 - 12x$ and say which is a maximum.

1. $dy/dx = 3x^2 - 12 = 0 \rightarrow x = \pm 2$.
2. $x = 2$: $y = 8 - 24 = -16$. $x = -2$: $y = -8 + 24 = 16$.
3. $d^2y/dx^2 = 6x$: at $x = 2$ it is 12 (minimum); at $x = -2$ it is -12 (maximum).

Maximum $(-2, 16)$; minimum $(2, -16)$

Common mistakes

Don't write	Write instead
 Differentiating $5x^3$ to $5x^2$	 Multiply by the power too: $15x^2$
 Leaving the constant in the derivative	 Constants differentiate to 0
 Finding y at a turning point from dy/dx	 Substitute into the original equation
 $7x$ differentiates to $7x^0$ left as $7x$	 $7x \rightarrow 7$

In the exam

- Write $dy/dx = \dots$ on its own line before substituting.
- State the coordinates of turning points, not just x .
- Say clearly 'maximum' or 'minimum' with your reason.

My notes